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SMA LOG

University of Rome Tor Vergata Italy
Department of Enterprise Engineering

KA2 - Cooperation for innovation and the exchange of good practices
Capacity Building in Higher Education
Joint project

Master in Smart Transport and Logistics for Cities / SmaLog
Batumi (Georgia), Sept. 13-14, 2018

1st Annual Training Seminar on SmaLog issues

Urban Goods Flow Modelling 2

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Urban goods flows modelling - 2: ***vehicle O/D matrices***

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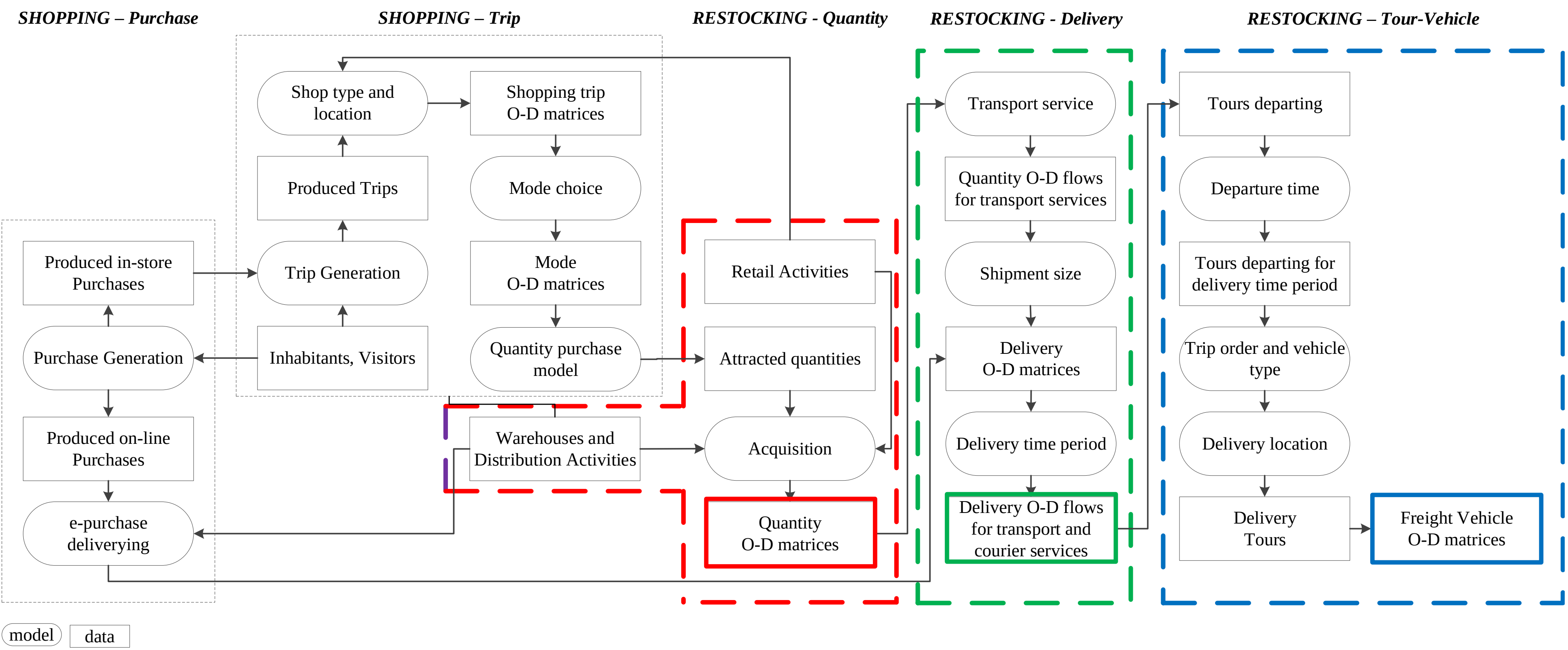
Demand model ⇒ O-D Matrices



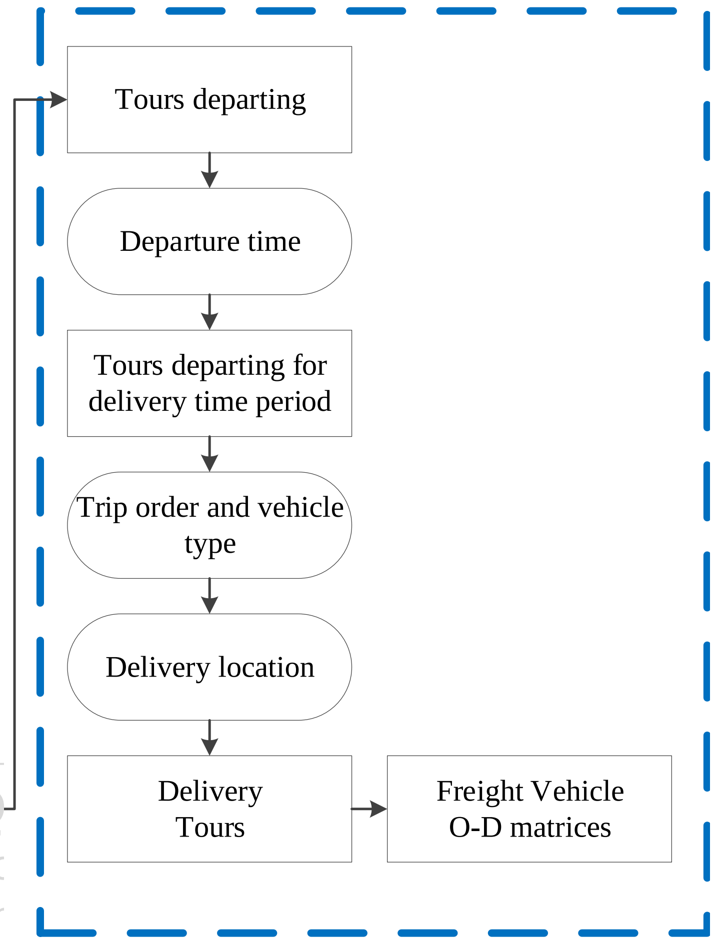
Quantity
allows to capture the mechanisms underlying the freight demand generation

Delivery
allows to follow the decisional and logistic process of restocking

Vehicle
Input for assignment and performance models

Demand model ⇒ Vehicle O-D Matrices

RESTOCKING – Tour-Vehicle



Q
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Vehicle O/D

Input for assignment and performance models

Vehicle *O-D* matrices

They give the average vehicle flow $\mathrm{VC}_{\mathrm{od}}^{\mathrm{sh}}\left[\mathrm{tvr}\right]$

transporting freight type *s* between the zone *o* and the zone *d* in a time period *h* characterized by:

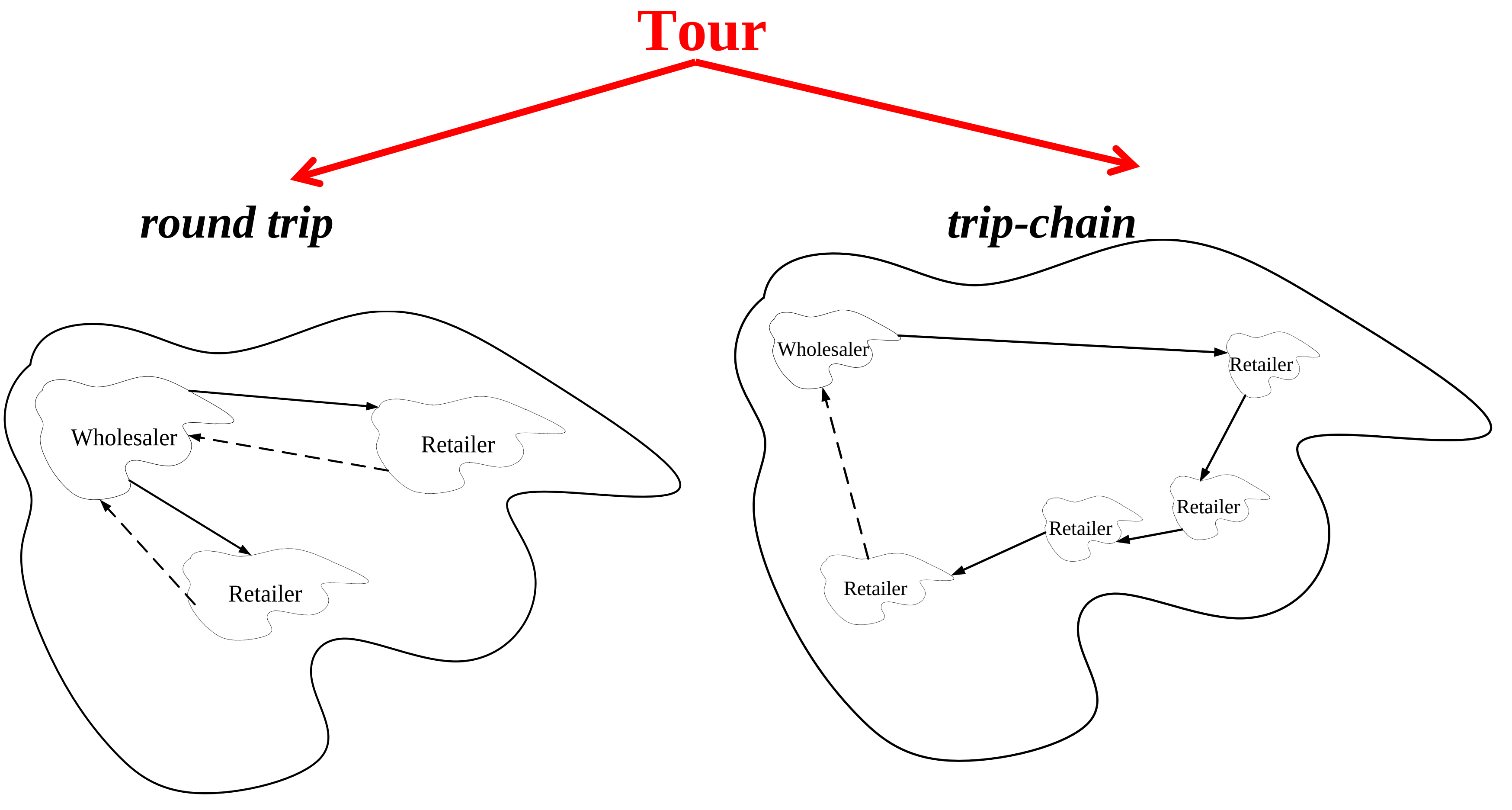
- service type (*r*)
- departure time from origin (*t*)
- vehicle type (*v*)

For simplicity of notation, the class index *s* (freight type) and *h* (time period) will be taken as understood unless otherwise stated $\Rightarrow$

$$\mathrm{VC}_{\mathrm{od}}\left[\mathrm{tvr}\right]$$

Vehicle O-D matrices

Delivery tour types



Freight vehicle O-D matrices

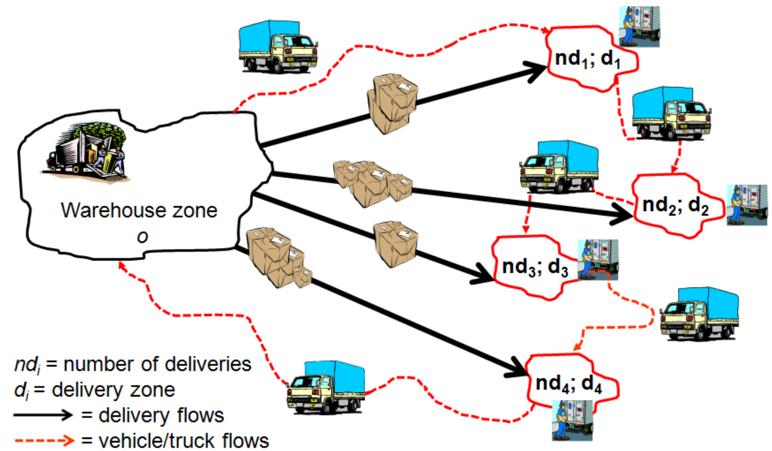
Problem definition

DELIVERIES

O-D	o	d_1	d_2	d_3	d_4
o		nd_1	nd_2	nd_3	nd_4
d_1					
d_2					
d_3					
d_4					

FREIGHT VEHICLES

O-D	o	d_1	d_2	d_3	d_4
o		1			
d_1			1		
d_2				1	
d_3					1
d_4	1				



- Restocker jointly chooses the **number** and the **location** of deliveries for each restocking tour
- Each restocker defines his tours trying to **reduce his costs** (e.g. using routing algorithm)
- The O-D matrices are the **sum** of **single** restocker choices

Freight vehicle O-D matrices

Delivery tour model

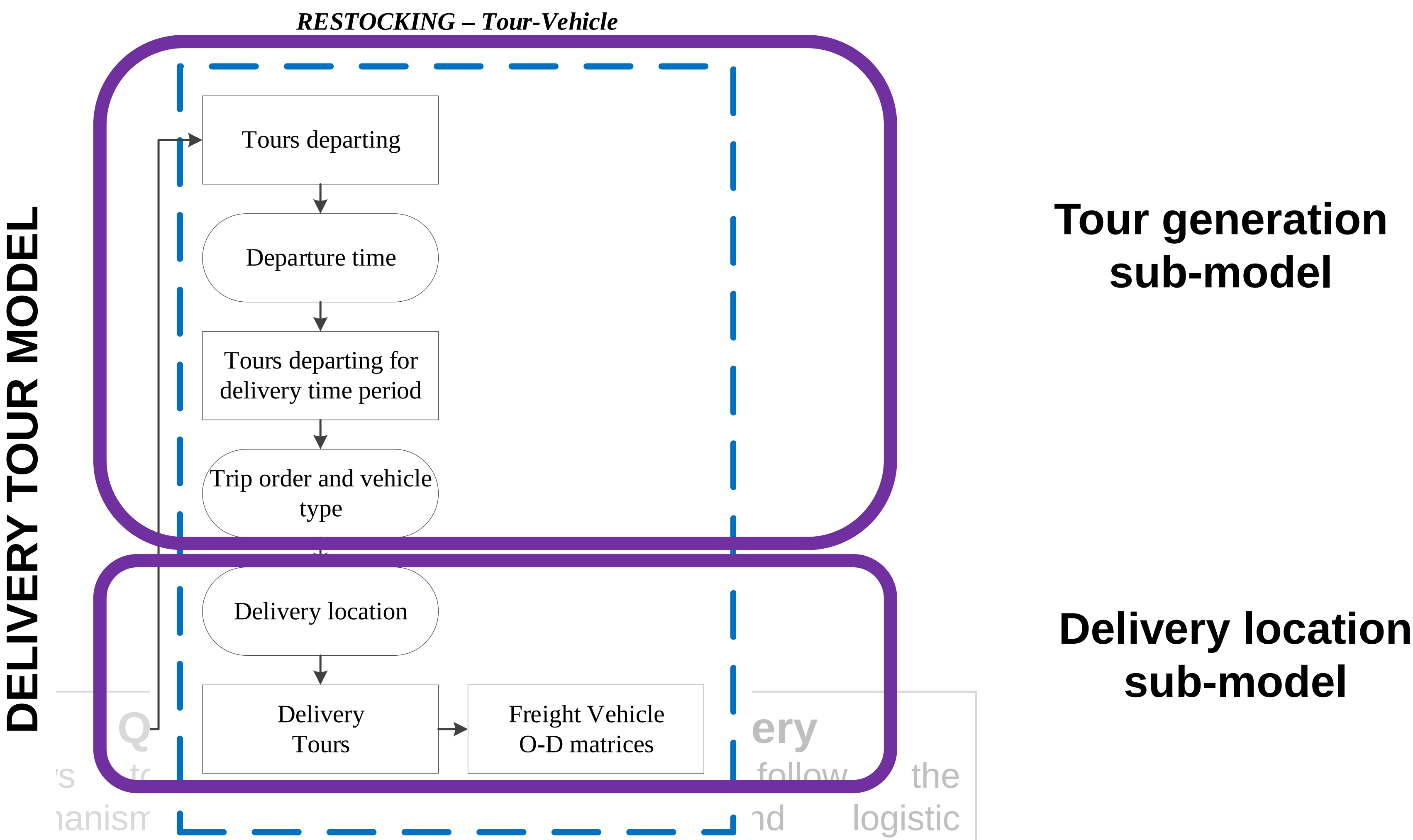
The freight vehicle O-D matrices can be obtained from the Delivery O-D matrices through the delivery tour model

Delivery O-D matrices → **Delivery tour model** → Freight vehicle O-D matrices

which uses a two-steps procedure:

- computation, from delivery O-D matrices, of the number of ***delivery tours departing from each zone o (tour generation sub-model)***
- definition of ***freight vehicle O-D matrices*** from delivery tours (***delivery location sub-model***)

Demand model ⇒ O-D Matrices



Delivery tour model

Tour generation sub-model

$$T_o[vntr] = T_o[r] \cdot p[t/ro] \cdot p[n/tro] \cdot p[v/tnro]$$

$$T_o[r] = \frac{ND_{o.}[r]}{\bar{n}} = \frac{ND_{o.}[r]}{\sum_n n \cdot p[n/tro]}$$

- ✓ T_o *[vntr]* is the number of tours departing (generated) from zone *o* at time *t* by transport service type *r* and vehicle type *v* with *n* stops/deliveries;
- ✓ T_o *[r]* is the number of tours by transport service type *r* departing (generated) from zone *o*;
- ✓ $ND_{o.}[r]$ is the average number of deliveries performed departing from origin zone *o*;
- ✓ *p[t/ro]* is the share or probability to undertake tour at time *t*;
- ✓ *p[n/tro]* is the share or probability to undertake tour with *n* stops/deliveries;
- ✓ *p[v/tnro]* is the share or probability to undertake tour by vehicle type *v*;
- ✓ $\bar{n}$ is the average number of deliveries performed by a tours departing from origin zone *o* at time *t;*

Delivery tour model

Trip chain order (number of stops) model

$$p[n / or] \qquad\qquad n = \{1, 2, \ldots\}$$

is the share or the probability that a tour departing from a given zone *o* has *n* stops/deliveries.

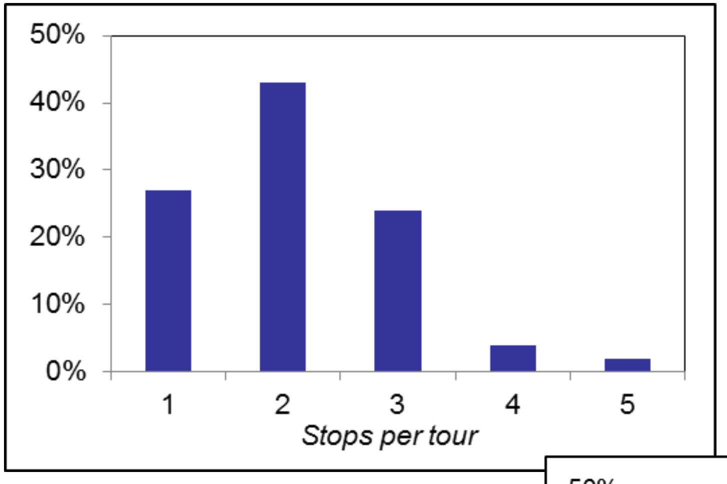
It allows to characterize the tours departing from a given warehouse zone (*o*) in terms of number of stops/deliveries.

Trip chain order distribution

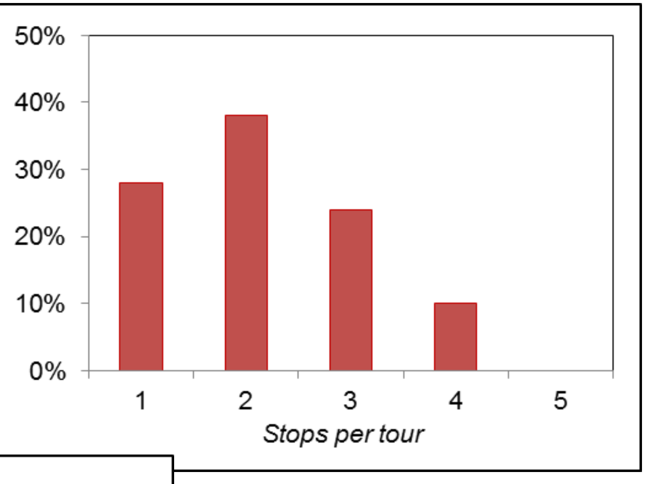
Revealed shares – wholesaler and carrier

$$p[n / ro]$$

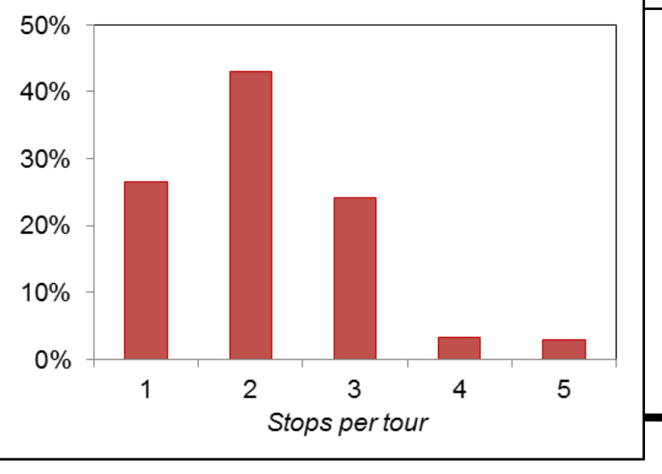
All freight types



Foodstuffs



Other goods



Delivery tour model

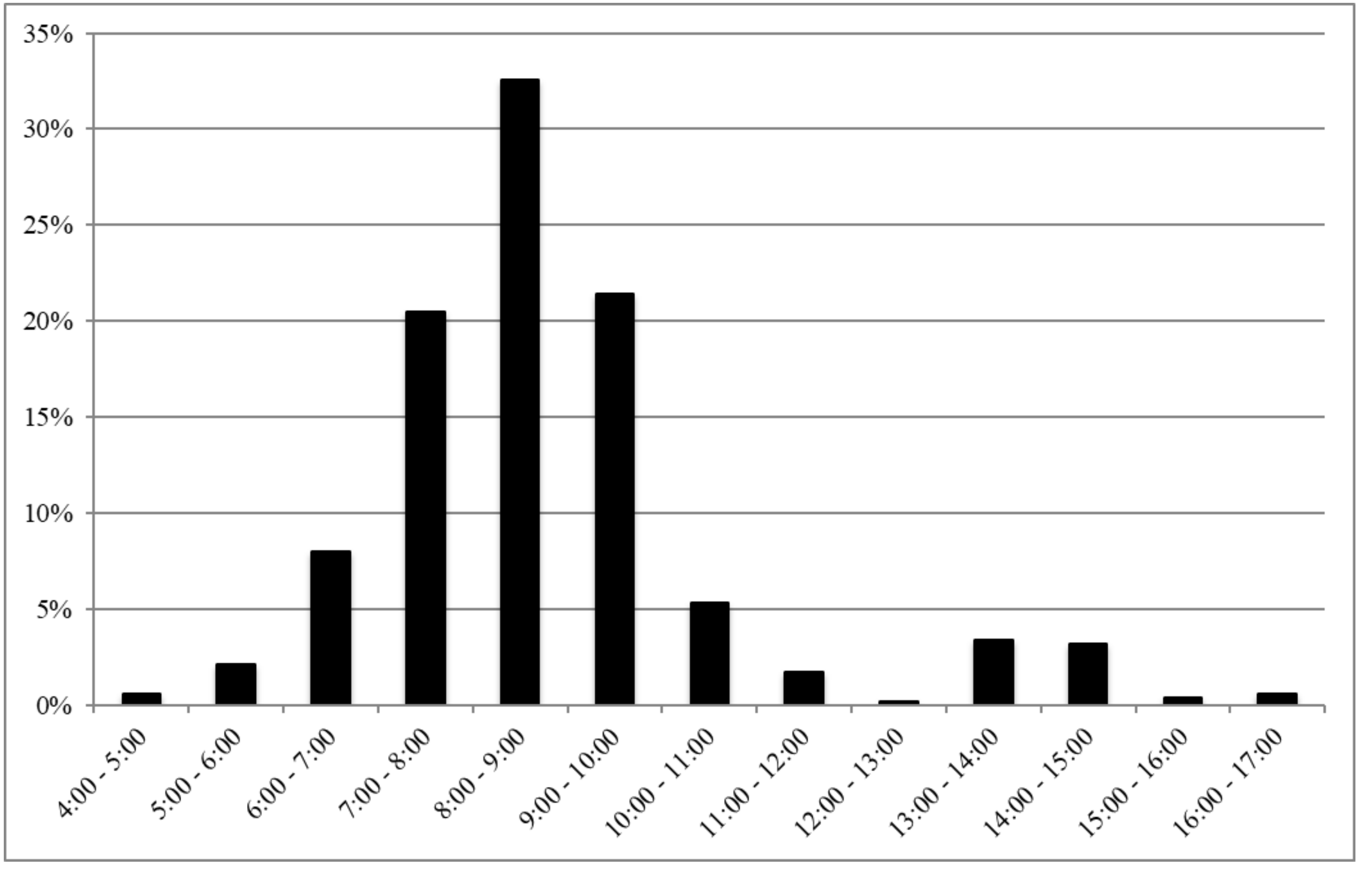
Departure time model

$$p[t/ro]$$

is the share or the probability to start tour from zone *o* at time *t*

Departure time model

Revealed shares



Vehicle type model

$$p[v / nro] \qquad v = \{LGV, MGV\}$$

is the share or the probability that a tour departing from a given zone *o* with *n* stops/deliveries, uses the vehicle of type *v*.

It allows us to characterize the tours departing from a given warehouse zone (*o*) in terms of type of vehicle used:

- *LGV = Light Goods Vehicle*
- *MGV = Medium Goods Vehicle*

Vehicle types

Light Goods Vehicle- LGV

Maximum load: ~ 700 kg
Load surface: ~ 2.5 mq

Maximum load: ~ 3,500 kg
Load surface: ~ 3.8 mq
Load volume: ~ 17.2 mc

Vehicle types

Heavy Goods Vehicle - HGV

<u>HGV ≤ 7.5 t</u>

Maximum load: ~ 7,000 kg

Load surface: ~ 8 mq

Load volume: ~ 20 mc

<u>HGV > 7.5 t</u>

Maximum load : ~ 26,000 kg

Load surface: ~ 10 mq

Load volume: ~ 25 mc

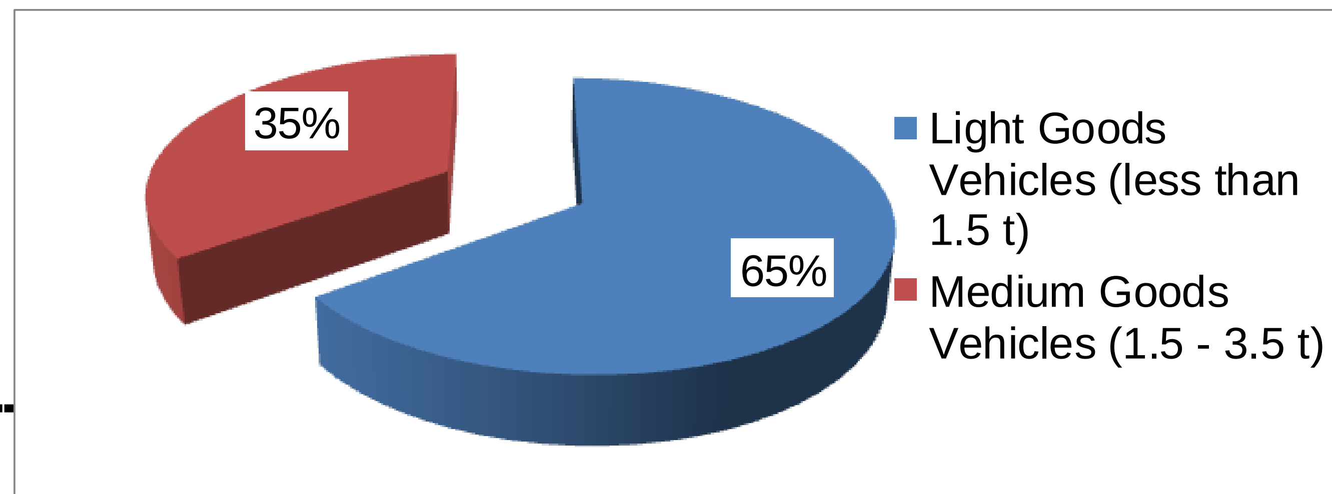
Vehicle type

Revealed shares

$$p[v / od]$$

	Foodstuffs	*Home accessories*
Light Goods Vehicle *(less than 1.5 tons)*	70%	51%
Heavy Goods Vehicle *(more than 1.5 tons)*	30%	49%
Total	**100%**	**100%**

- ✓ Average transported quantity:
  - ➢ LGV (< 1.5 t): 800 kg
  - ➢ MGV (1.5 – 3.5 t): 1,716 kg



Roma (2008)

Delivery location models

✓ *Two main approaches:*

- disaggregate: routing-based algorithms

- aggregate: partial shares

Vehicle O-D matrices

Delivery location sub-model (partial shares)

Number of tours
departing from a given zone *o* using vehicle type *v*

$$T_o[vtn] = T_o[tn] \cdot p[v / no]$$

Sequence of stop locations per tour

$$p\left[d_j^{k+1} / d_i^k vno\right]$$

Vehicle O-D matrices

$$VC_{d_i d_j}[vno]$$

Delivery tour model

Delivery location model

$$p\left[d_j^{k+1} / d_i^k \; vno\right] \qquad\qquad k = \{0,\ 1,\ 2, \ldots\}$$

is the share or the probability of delivering in zone d_j the delivery *(k+1)* conditional to have previously delivered in zone d_i the delivery *k*, within a tour with *n* stops/deliveries departing from a given zone *o and using a* vehicle of type *v.*

It allows us to define the sequence of zones delivered during the tour.

Examples of delivery tour models

Vehicle O-D matrices

Structure of delivery tour model

delivery O-D matrices $\qquad ND_{od}$

↓

Number of tours departing from a given zone *o* with *n* stops $\qquad T_o[n] = ND_{o.}/\bar{n} \cdot p[n/o]$

↓

Number of tours departing from a given zone *o* at time *t* $\qquad T_o[tn] = T_o[n] \cdot p[t/no]$

↓

Number of tours departing from a given zone *o* using vehicle type *v* $\qquad T_o[vtn] = T_o[tn] \cdot p[v/no]$

↓

Sequence of stop locations per tour $\qquad p\left[d_j^{k+1} / d_i^k vno\right]$

↓

Vehicle O-D matrices $\qquad VC_{d_i d_j}[vno]$

Delivery tour model

Structure

delivery O-D matrices

$$ND_{od}$$

Number of tours departing from a given zone *o* with *n* stops

$$T_o[n] = ND_{o.} / \bar{n} \cdot p[n/o]$$

Number of tours
departing from a given zone *o* at time *t*

$$T_o[tn] = T_o[n] \cdot p[t/no]$$

Number of tours
departing from a given zone *o* using vehicle type *v*

$$T_o[vtn] = T_o[tn] \cdot p[v/no]$$

Sequence of stop locations per tour

$$p\left[d_j^{k+1} / d_i^k vno\right]$$

Vehicle O-D matrices

$$VC_{d_i d_j}[vno]$$

Trip chain order distribution

$$p[n/tro]$$

p[n/tro] is the probability that deliveries are performed by tours departing from a given zone *o* with *n* stops/deliveries; it can be expressed by a logit model:

$$p[n/tro] = \frac{\exp(V_n)}{\sum_{n'} \exp(V_{n'})}$$

$$n \in I = \{number\ of\ stops\ /\ deliveries\}$$

$$V_n = \Sigma_j \beta_j X_{jn}$$

X_{jn} is the *j*-th attribute related to have tour with *n* stops/deliveries.

Example of trip chain order distribution model

Wholesaler in own account

The systemic utility, V_n, has been specified as linear combination of the following attributes:

- *VEH*, dummy variable equal to *1* if the used vehicle is a Light Goods Vehicle, *0* otherwise;
- *q*, average quantity of freight delivered at each stop (i.e. delivery point) along the journey, expressed in tons;
- *FGT*, dummy variable equal to *1* if the delivered freight belongs to the *foodstuffs* class, *0* otherwise;
- IAA_o, *retailer* accessibility index of zone *o*, from which the tour departs (e.g. warehouse location):

$$IAA_o = \left[AA_o - \min_z \left(AA_z \right) \right] \Big/ \left[\max_z \left(AA_z \right) - \min_z \left(AA_z \right) \right]$$

where AA_x is the accessibility of zone *x* estimated as:

$$AA_x = \sum_j \left(UL_j \right)^{6.334} \cdot \exp\left[-3.913 \cdot dist_{xj} \right]$$

- ✓ UL_j the number of retail establishments of zone *j* to be restocked,
- ✓ $dist_{xj}$ the distance between zone *x* and *j*,
- ✓ α_1 and α_2 calibration parameters,

Example of trip chain order distribution model

Wholesaler in own account

ρ^2 = 0.18

$$p[n/to] = \exp(V_n) \Big/ \sum_{n'} \exp(V_{n'}) \quad n \in I = \{1stop, 2stops, 3stops, > 3stops\}$$

$$V_1 = \underset{(2.5)}{2.430} + \underset{(1.6)}{0.252} \cdot VEH$$

$$V_2 = -\underset{(-1.1)}{0.399} \cdot VEH - \underset{(-2.1)}{0.075} \cdot \ln(IAA_o) - \underset{(-1.7)}{0.151} \cdot q + \underset{(2.1)}{0.965} \cdot FGT + \underset{(2.6)}{2.429}$$

$$V_3 = -\underset{(-1.1)}{0.042} \cdot \ln(IAA_o) + \underset{(2.7)}{1.381} \cdot FGT + \underset{(1.8)}{1.788}$$

$$V_{>3} = -\underset{(-1.5)}{0.799} \cdot VEH - \underset{(-1.4)}{0.082} \cdot \ln(IAA_o) - \underset{(-1.5)}{0.195} \cdot q + \underset{(3.1)}{2.512} \cdot FGT$$

light goods vehicles have **tours** with **few stops/deliveries**	**increasing** the accessibility of restocking zone, the number of stops/deliveries **decreases**	**more** stops/**deliveries** per tour for **small delivered quantity**	**more** stops/deliveries for **foodstuffs**

Delivery tour model

Structure

delivery O-D matrices

$$ND_{od}$$

↓

Number of tours departing from a given zone *o* with *n* stops

$$T_o[n] = ND_{o.}/\bar{n} \cdot p[n/o]$$

↓

Number of tours departing from a given zone *o* at time *t*

$$T_o[tn] = T_o[n] \cdot p[t/no]$$

↓

Number of tours departing from a given zone *o* using vehicle type *v*

$$T_o[vtn] = T_o[tn] \cdot p[v/no]$$

↓

Sequence of stop locations per tour

$$p\left[d_j^{k+1} / d_i^k vno\right]$$

↓

Vehicle O-D matrices

$$VC_{d_i d_j}[vno]$$

Vehicle type model

$$p\left[v / nto\right]$$

p[v/nto] is the probability to restock by vehicle type *v*; it can be expressed by a logit model:

$$p\left[v / nto\right] = \frac{\exp(\alpha V_v)}{\sum_{v'} \exp(\alpha V_{v'})} \qquad V_v = \Sigma_j \beta_j X_{jv}$$

X_{jv} is the *j*-th attribute related to vehicle type *v* .

Vehicle type model

Retailer in own account

$$V_{car} = \underset{(3.4)}{1.01}$$

$$V_{lgv} = \underset{(1.0)}{0.003} \cdot EM + \underset{(2.2)}{0.50} \cdot q + \underset{(3.4)}{0.05}$$

$$V_{mgv} = \underset{(2.2)}{0.004} \cdot EM + \underset{(3.4)}{3.00} \cdot q + \underset{(1.4)}{0.006} \cdot STORE$$

ρ^2 = 0.38

where

- V_{car} is the systematic utility for using the ***car*** (e.g. SUV, station-wagon, pick-up),
- V_{lgv} is the systematic utility for using the ***Light Goods Vehicle*** (<1.5 ton),
- V_{mgv} is the systematic utility for using the ***Medium Goods Vehicle*** (1.5 – 3.5 tons),
- $STORE$ is the surface of store, expressed in m^2,
- EM is the number of employees at shop to be restocked,
- q is the average *shipment size*, expressed in tons.

Vehicle type model

Retailer in own account

$$V_{car} = \underset{(3.4)}{1.01}$$

$$V_{lgv} = \underset{(1.0)}{0.003} \cdot EM + \underset{(2.2)}{0.50} \cdot q + \underset{(3.4)}{0.05}$$

$$V_{mgv} = \underset{(2.2)}{0.004} \cdot EM + \underset{(3.4)}{3.00} \cdot q + \underset{(1.4)}{0.006} \cdot STORE$$

ρ^2 = 0.38

where

- V_{car} is the systematic utility for using the ***car*** (e.g. SUV, station-wagon, pick-up),
- V_{lgv} is the systematic utility for using the ***Light Goods Vehicle*** (<1.5 ton),
- V_{mgv} is the systematic utility for using the … (1.5 – 3.5 tons),
- *STORE* … expressed …
- *EM* is … t shop to be …
- *q* is th… pressed in tons.

Retailer tends to use **car** for **small shops** with few employees and a **small depot**

The type of vehicle depends on average shipment size

Delivery tour model

Structure

delivery O-D matrices

$$ND_{od}$$

Number of tours departing from a given zone *o* with *n* stops

$$T_o[n] = ND_{o.} / \bar{n} \cdot p[n / o]$$

Number of tours departing from a given zone *o* at time *t*

$$T_o[tn] = T_o[n] \cdot p[t / no]$$

Number of tours departing from a given zone *o* using vehicle type *v*

$$T_o[vtn] = T_o[tn] \cdot p[v / no]$$

Sequence of stop locations per tour

$$p\left[d_j^{k+1} / d_i^k vno\right]$$

Vehicle O-D matrices

$$VC_{d_i d_j}[vno]$$

Delivery location model

$$p\left[d_j^{k+1} / d_i^k \, vno\right] = \exp\left(V_{d_j^{k+1}}\right) \Big/ \sum_{d'} \exp\left(V_{d'}\right)$$

$p\left[d_j^{k+1} / d_i^k \, vnro\right]$ is the probability of delivering in zone d_j the delivery *(k+1)* conditioned to have previously delivered in zone d_i the delivery k within a tour with n deliveries that departs from zone o,

$V_{d_j^{k+1}}$ is the systematic utility of delivering in zone d_j the delivery *(k+1)*

The survey has revealed that different behaviors could be followed by a restocker in the choice of first destination within a tour and the following ones

- **Choice of first delivery location,**
- **Choice of following delivery locations**

Delivery location model

✓ **Wholesaler in own account and carrier:**
first delivery location

$\rho^2 = 0.33$

$$V_{d_j^1} = 0.213 \cdot \ln\left(AD_{d_j^1}\right) - 0.028 \cdot dist_{od_j^1} + 2.03 \cdot DS_{od_j^1} + 7.84 \cdot IAA_{d_j^1}$$

- AD_{dj} is the number of retail employees in zone d_j;
- $dist_{odj}$ is the distance between zone o and d_j, expressed in km;
- DS_{odj} is the share of deliveries on od_j pair respect to all deliveries departing from zone o;
- IAA_{dj} is the retailer accessibility of zone d_j.

Delivery location model

✓ **Wholesaler in own account and carrier:**
first delivery location

ρ^2 = 0.33

$$V_{d_j^1} = 0.213 \cdot \ln\left(AD_{d_j^1}\right) - 0.028 \cdot dist_{od_j^1} + 2.03 \cdot DS_{od_j^1} + 7.84 \cdot IAA_{d_j^1}$$

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- DS_{odj} is the share of deliveries on od_j pair respect to all deliveries departing from zone o;
- IAA_{dj} is the retailer ac

The probability of a zone increases with its attraction capacity

The probability of a zone increases with its accessibility and closeness

Delivery location model

✓ **Wholesaler in own account and carrier:**
next delivery locations

$\rho^2 = 0.25$

$$V_{d_j^{k+1}} = 0.291 \cdot \ln\left(AD_{d_j^{k+1}}\right) + 8.408 \cdot DS_{od_j^{k+1}} - 0.325 \cdot dist_{d_i^k d_j^{k+1}} +$$
$$-1.655 \cdot \ln\left(HT_{d_j^{k+1}}\right) + 1.064 \cdot ASA_{d_j^{k+1} = d_i^k}$$

- AD_{dj} is the number of retail employees in zone d_j;
- $dist_{didj}$ is the distance between zone *o* and d_j, expressed in km;
- DS_{odj} is the share of deliveries on od_j pair respect to all deliveries departing from zone *o*;
- HT_{dj} is the ratio between the distance to be covered to reach the next delivery location and the current covered distance.

Delivery location model

✓ **Wholesaler in own account and carrier:**
next delivery locations

ρ² = 0.25

$$V_{d_j^{k+1}} = 0.291 \cdot \ln\left(AD_{d_j^{k+1}}\right) + 8.408 \cdot DS_{od_j^{k+1}} - 0.325 \cdot dist_{d_i^k d_j^{k+1}} +$$

$$-1.655 \cdot \ln\left(HT_{d_j^{k+1}}\right) + 1.064 \cdot ASA_{d_j^{k+1}=d_i^k}$$

- AD_{dj} is the number of retail employees in zone d_j;
- $dist_{didj}$ is the distance between zone o and d_j, expressed in km;
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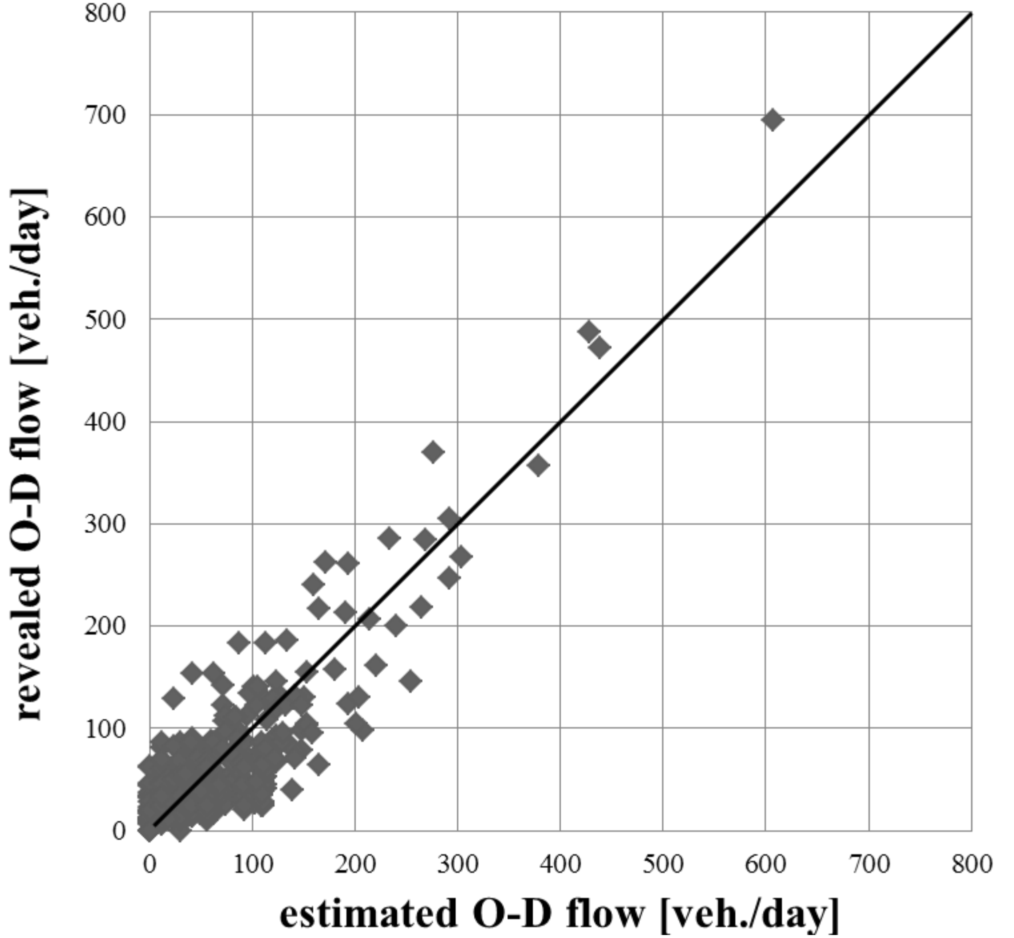
- HT_{dj} … he distance to be covered to r… ry … covered distance.

The probability of a zone increases with its attraction capacity

The probability of a zone increases with its closeness

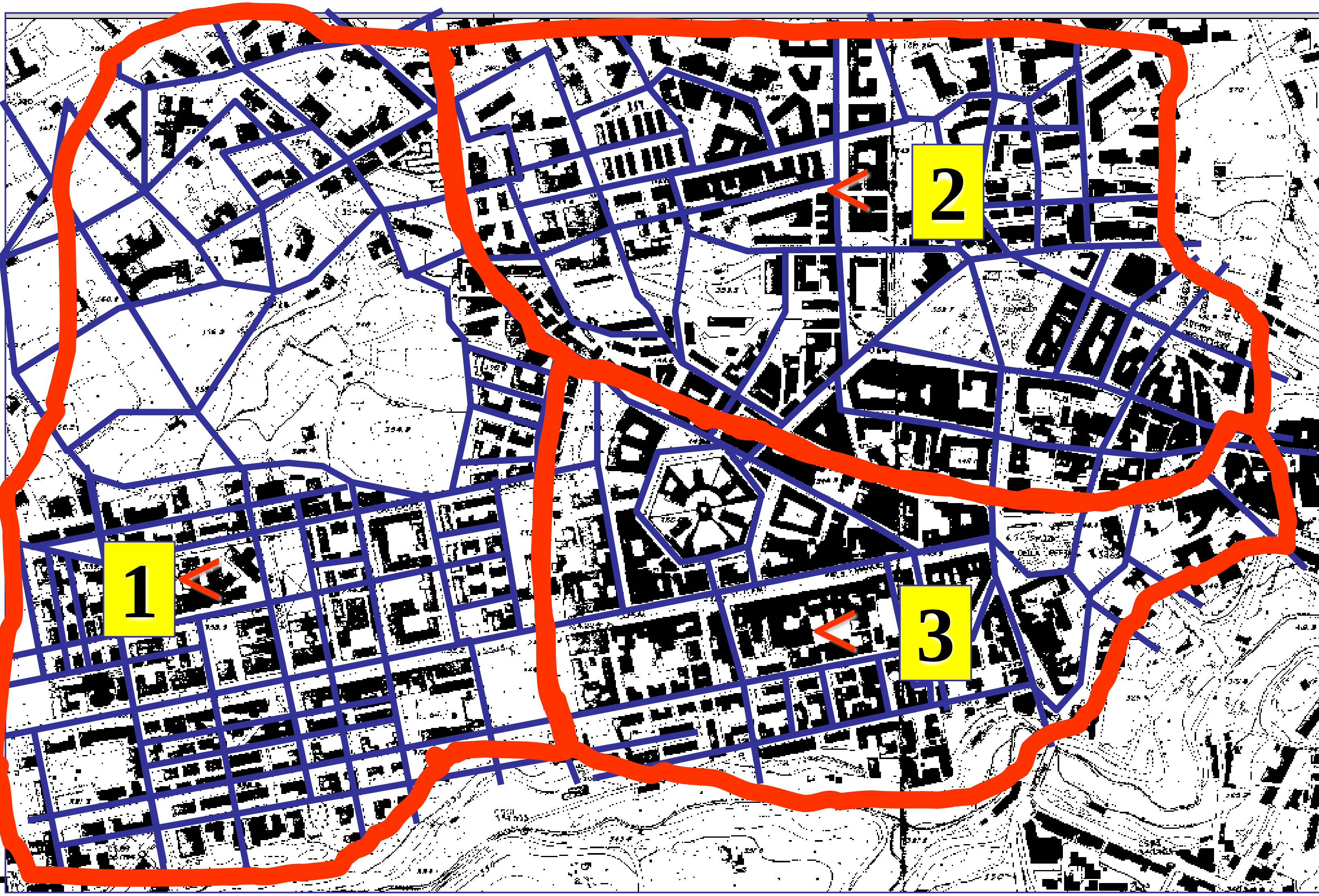
Revealed vs Estimated Freight Vehicle O-D flows (ZTL in Rome)



Revealed *vs* Estimated O-D vehicle flows

Exercise

Traffic zones



Vehicle O-D matrices

Tour departure (t)
- departing at 10:30 in time slice before 11am
- departing at 11:30 in time slice after 11am

Number of stops (n)
- Maximum 3 stops

Vehicle type (v)
- One vehicle type

Sequence of stops (k)

Vehicle O-D matrices

Stops per tour

r = Retailer o. a.

	num =1	*num =2*	*num =3*	**average**
Zone 1	50%	20%	30%	**33%**
Zone 2	40%	20%	40%	**33%**
Zone 3	60%	30%	10%	**33%**
average	**50%**	**23%**	**27%**	**33%**

r = Other

	num =1	*num =2*	*num =3*	**average**
Zone 1	10%	70%	20%	**33%**
Zone 2	20%	60%	20%	**33%**
Zone 3	40%	50%	10%	**33%**
average	**23%**	**60%**	**17%**	**33%**

$$\bar{n}[ro] = \sum_n n \cdot p[n / ro]$$

	$\bar{n}[ret]$	$\bar{n}[\mathrm{oth}]$
Zone 1	1.8	2.1
Zone 2	2.0	2.0
Zone 3	1.5	1.7

Vehicle O-D matrices

Tour departing from each zone

$$T_o\left[t\tau r\right] = ND_{o.}\left[t\tau r\right] / \bar{n}$$

r = Retailer o. a.

	T_o [departing at 10:30 am]
Zone 1	8.3
Zone 2	78.3
Zone 3	35.1
Total	**121.7**

	T_o [departing at 11:30 am]
Zone 1	52.2
Zone 2	575.5
Zone 3	227.7
Total	**855.4**

r = Other

	T_o [departing at 10:30 am]
Zone 1	3.6
Zone 2	33.6
Zone 3	15.0
Total	**52.2**

	T_o [departing at 11:30 am]
Zone 1	22.4
Zone 2	246.6
Zone 3	97.6
Total	**366.6**

Vehicle O-D matrices

Tour departing from each zone and num. of stops

$$T_o\left[vn\tau r\right]=T_o\left[t\tau r\right]\cdot p\left[n/t\tau ro\right]$$

r = Retailer o. a.

at 10:30 am				
	To[n=1]	To[n=2]	To[n=3]	**Total**
Zone 1	4.2	1.7	2.5	**8.4**
Zone 2	31.3	15.7	31.3	**78.3**
Zone 3	21.1	10.5	3.5	**35.1**
Total	**56.6**	**27.9**	**37.3**	**121.8**

at 11:30 am				
	To[n=1]	To[n=2]	To[n=3]	**Total**
Zone 1	5.2	36.5	10.4	**52.1**
Zone 2	115.1	345.3	115.1	**575.5**
Zone 3	91.1	113.9	22.8	**227.8**
Total	**211.4**	**495.7**	**148.3**	**855.4**

r = Other

at 10:30 am				
	To[n=1]	To[n=2]	To[n=3]	**Total**
Zone 1	1.8	0.7	1.1	**3.6**
Zone 2	13.4	6.7	13.4	**33.5**
Zone 3	9.0	4.5	1.5	**15.0**
Total	**24.2**	**11.9**	**16.0**	**52.1**

at 11:30 am				
	To[n=1]	To[n=2]	To[n=3]	**Total**
Zone 1	2.2	15.7	4.5	**22.4**
Zone 2	49.3	148.0	49.3	**246.6**
Zone 3	39.0	48.8	9.8	**97.6**
Total	**90.5**	**212.5**	**63.6**	**366.6**

Vehicle O-D matrix procedure

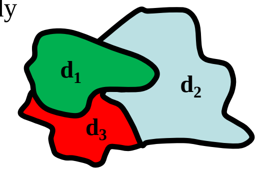
Iterative Algorithm

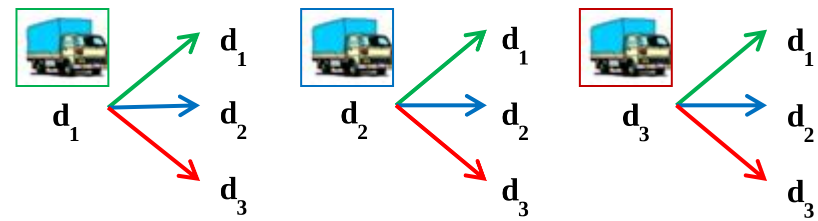
Example:

- 3 traffic zones
- 2 stops matrices → 2 iterations

Hp:

- One delivery → One stop → One trip





The 1° iteration: calculation of the first trips of tours

Tour

T_o [n=2]	
d_1	100
d_2	200
d_3	150

Matrices of the 1° stop

$p[d^1_j/d^0_i]$	d_1	d_2	d_3	tot
d_1	20%	50%	30%	100%
d_2	60%	20%	20%	100%
d_3	40%	30%	30%	100%

Vehicle matrice

O-D	d_1	d_2	d_3	tot
d_1	20	50	30	100
d_2	120	40	40	200
d_3	60	45	45	150
tot	*200*	*135*	*115*	450

Vehicle of the 1°stop

T^1_o [n=2]	
d_1	*200*
d_2	*135*
d_3	*115*

Split of vehicle trips

Vehicles arriving to the 1° destination

Vehicle O-D matrix procedure

Iterative Algorithm 2/2

The 2° iteration: calculation of the successive trips of tours

Tour

T^1_o [n=2]	
d_1	200
d_2	135
d_3	115

Matrices of the 2° stop

$p[d^2_j/d^1_i]$	d_1	d_2	d_3	tot
d_1	30%	40%	30%	100%
d_2	40%	40%	20%	100%
d_3	20%	60%	20%	100%

Split of vehicle trips

Vehicle matrice

O-D	d_1	d_2	d_3	tot
d_1	60	80	60	200
d_2	54	54	27	135
d_3	23	69	23	115
tot	*137*	*203*	*110*	450

Vehicle of the 2°stop

T^2_o [n=2]	
d_1	*137*
d_2	*203*
d_3	*110*

Vehicles arriving to the 2° destination

O-D VC	d_1	d_2	d_3	tot
d_1	20	50	30	100
d_2	120	40	40	200
d_3	60	45	45	150
tot	*200*	*135*	*115*	450

+

O-D VC	d_1	d_2	d_3	tot
d_1	60	80	60	200
d_2	54	54	27	135
d_3	23	69	23	115
tot	*137*	*203*	*110*	450

=

FINAL Vehicle O -D matrix

O-D VC	d_1	d_2	d_3	tot
d_1	80	130	90	300
d_2	174	94	67	335
d_3	83	114	68	265
tot	337	338	225	900

Vehicle O-D matrices

Choice of vehicle type and delivery zone

$$T_o[vnt\tau r] = T_o[t\tau r] \cdot p[n / t\tau ro] \cdot p[v / n\tau ro]$$

= 1

$$p\left[d_j^{k+1} / d_i^k \right]$$

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	20%	50%	30%	**100%**
Zone 2	60%	20%	20%	**100%**
Zone 3	40%	30%	30%	**100%**

Vehicle O-D matrices

Retailer o. a. , departing at 10:30 am, n = 1 stop

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	0.8	2.1	1.3	**4.2**
Zone 2	18.8	6.3	6.3	**31.3**
Zone 3	8.4	6.3	6.3	**21.1**
Total	**28.1**	**14.7**	**13.9**	**56.6**

Vehicle O-D matrices

Retailer, departing at 10:30 am, n = 2 stops

1° *Iteration*

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	0.3	0.9	0.5	**1.7**
Zone 2	9.4	3.1	3.1	**15.7**
Zone 3	4.2	3.2	3.2	**10.5**
Total	**14.0**	**7.1**	**6.8**	**27.9**

2° *Iteration*

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	2.8	7.0	4.2	**14.0**
Zone 2	4.3	1.4	1.4	**7.1**
Zone 3	2.7	2.0	2.0	**6.8**
Total	**9.8**	**10.4**	**7.7**	**27.9**

Total **Full**

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	3.1	7.8	4.7	**15.7**
Zone 2	13.7	4.6	4.6	**22.8**
Zone 3	6.9	5.2	5.2	**17.3**
Total	**23.8**	**17.6**	**14.5**	**55.8**

Vehicle O-D matrices

Retailer, departing at 10:30 am, n = 3 stops

1° *Iteration*

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	0.5	1.3	0.8	**2.5**
Zone 2	18.8	6.3	6.3	**31.3**
Zone 3	1.4	1.1	1.1	**3.5**
Total	**20.7**	**8.6**	**8.1**	**37.3**

2° *Iteration*

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	4.1	10.3	6.2	**20.7**
Zone 2	5.1	1.7	1.7	**8.6**
Zone 3	3.2	2.4	2.4	**8.1**
Total	**12.5**	**14.5**	**10.3**	**37.3**

3° *Iteration*

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	2.5	6.2	3.7	**12.5**
Zone 2	8.7	2.9	2.9	**14.5**
Zone 3	4.1	3.1	3.1	**10.3**
Total	**15.3**	**12.2**	**9.7**	**37.3**

Total **Full**

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	7.1	17.8	10.7	**35.7**
Zone 2	32.6	10.9	10.9	**54.3**
Zone 3	8.8	6.6	6.6	**21.9**
Total	**48.5**	**35.3**	**28.1**	**111.9**

Vehicle O-D matrices

departing at 10:30 am

Retailer o. a.

Full	*Zone 1*	*Zone 2*	*Zone 3*	**Total**
Zone 1	11.1	27.8	16.7	**55.5**
Zone 2	65.1	21.7	21.7	**108.5**
Zone 3	24.1	18.1	18.1	**60.3**
Total	**100.3**	**67.6**	**56.4**	**224.3**

Other

Full	*Zone 1*	*Zone 2*	*Zone 3*	**Total**
Zone 1	88.5	221.3	132.8	**442.7**
Zone 2	473.4	157.8	157.8	**789.1**
Zone 3	166.4	124.8	124.8	**415.9**
Total	**728.4**	**503.9**	**415.4**	**1.647.7**

Vehicle O-D matrices

departing at 11:30 am

Retailer o. a.

Full

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	4.8	11.9	7.1	**23.8**
Zone 2	27.9	9.3	9.3	**46.4**
Zone 3	10.3	7.7	7.7	**25.8**
Total	**42.9**	**28.9**	**24.2**	**96.0**

Other

Full

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	38.0	94.9	56.9	**189.8**
Zone 2	202.9	67.6	67.6	**338.2**
Zone 3	71.3	53.5	53.5	**178.3**
Total	**312.2**	**216.0**	**178.1**	**706.3**

Total Vehicle O-D matrices

departing at 10:30 am

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	99.6	249.1	149.5	**498.2**
Zone 2	538.5	179.5	179.5	**897.5**
Zone 3	190.5	142.9	142.9	**476.2**
Total	**828.7**	**571.5**	**471.8**	**1.872.0**

departing at 11:30 am

	Zone 1	*Zone 2*	*Zone 3*	**Total**
Zone 1	42.7	106.8	64.1	**213.6**
Zone 2	230.8	76.9	76.9	**384.6**
Zone 3	81.6	61.2	61.2	**204.1**
Total	**355.1**	**245.0**	**202.2**	**802.3**