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SMA LOG

University of Rome Tor Vergata Italy
Department of Enterprise Engineering

KA2 - Cooperation for innovation and the exchange of good practices

Capacity Building in Higher Education

Joint project

Master in Smart Transport and Logistics for Cities / SmaLog

Batumi (Georgia), Sept. 13-14, 2018

1st Annual Training Seminar on SmaLog issues

Transportation Supply Models

road networks and transit systems

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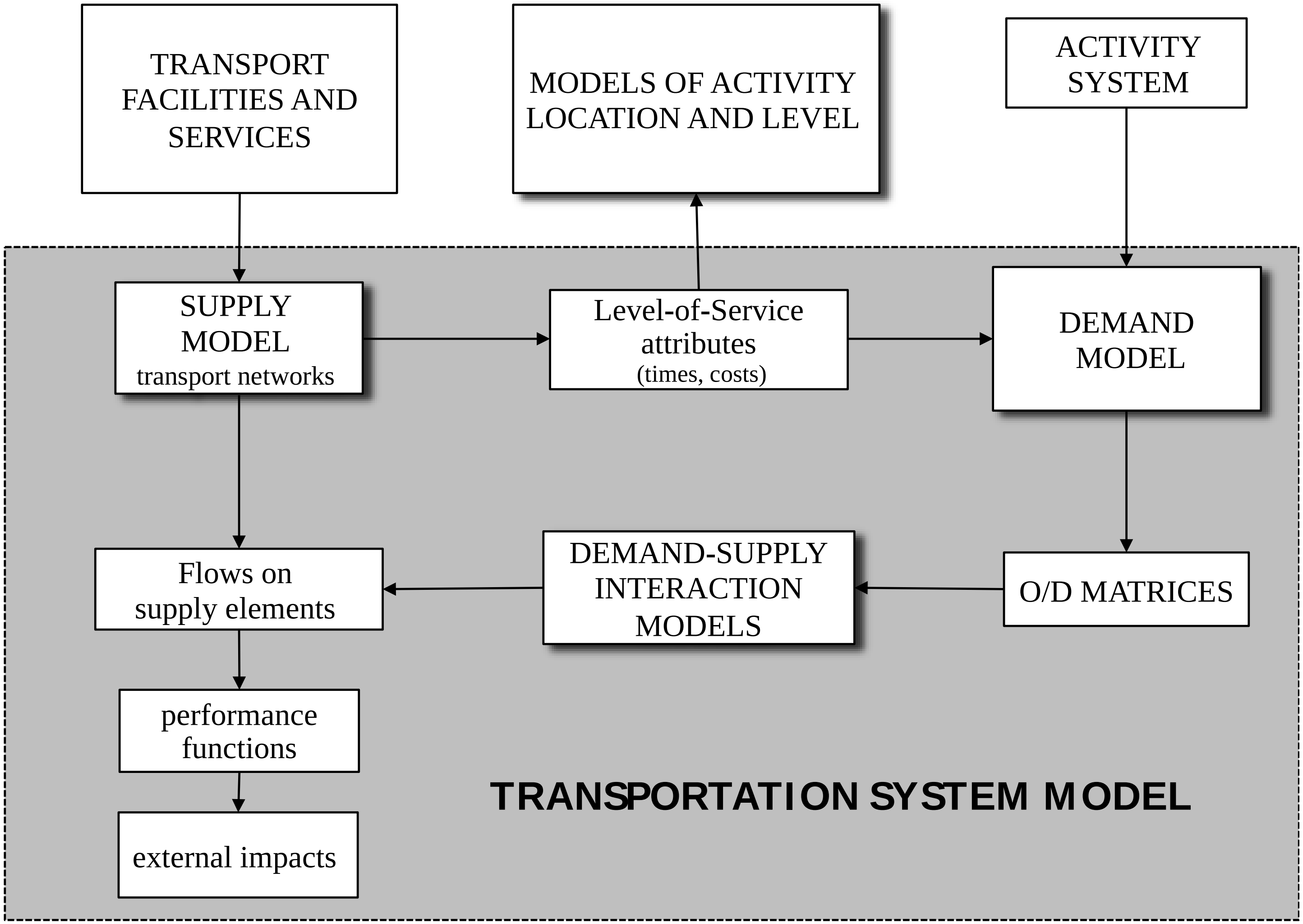
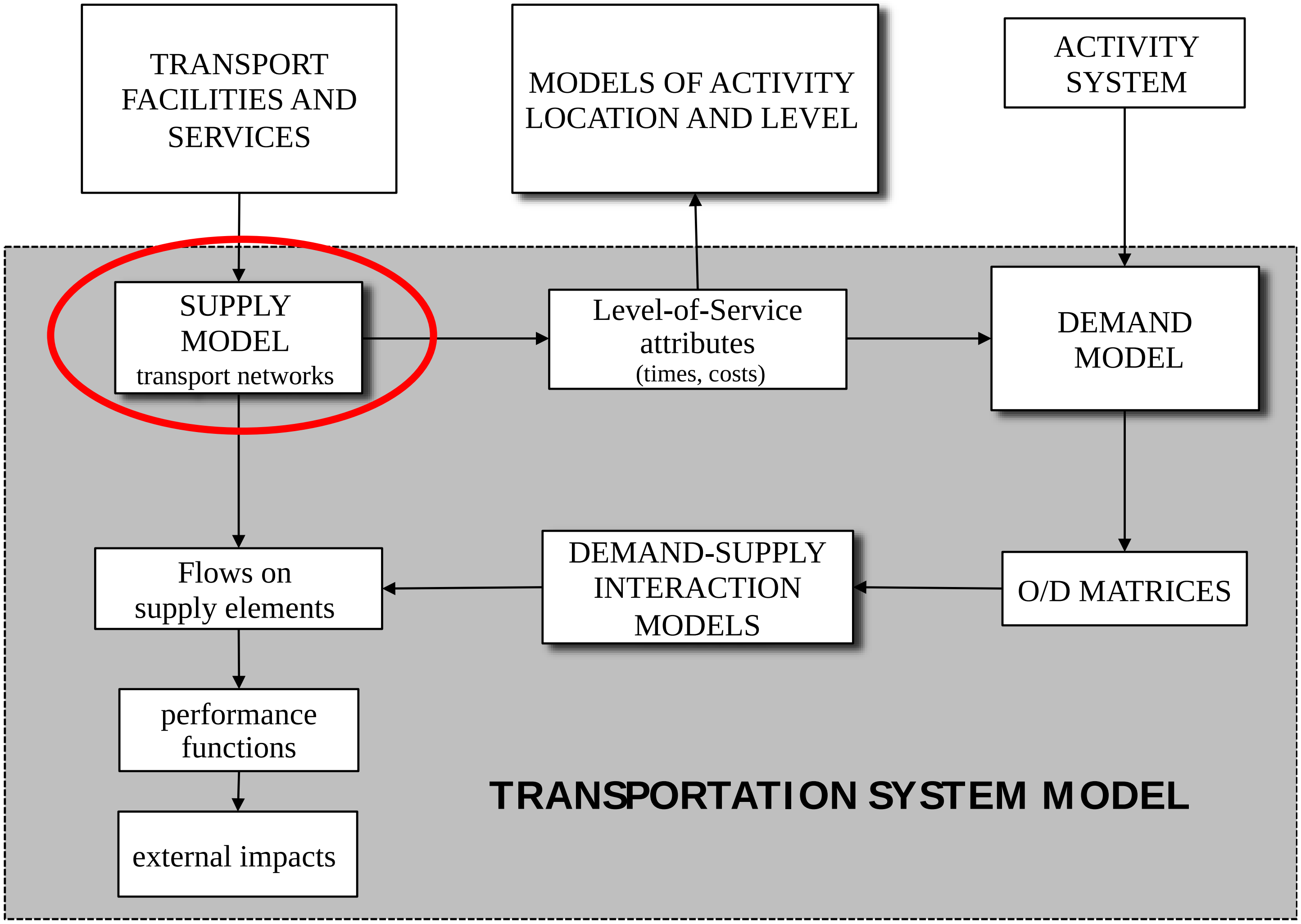
Transport modelling

General structure of simulation models



Transport modelling
General structure of simulation models



Supply models

- Through the simulation of supply facilities functioning, they allow the estimation of supply performances (i.e. level-of-service attributes), like:
  - travel times
  - speeds
  - impacts (e.g. pollutant emissions)

  …

- Given path flows, supply models allow the estimation of flows on supply facilities (links, nodes, …)

Graph theory

Graph $G \equiv (N,L)$: is defined as an ordered pair of sets: N, the set of elements known as nodes or vertices, and $L \subseteq N \times N$, a set of pairs of nodes belonging to N, known as links or arcs.

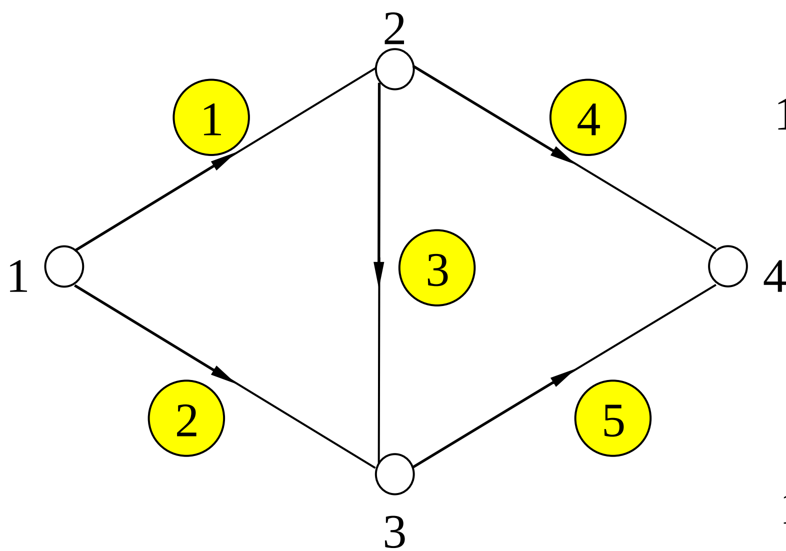
Transportation graphs are usually **oriented**.

A link connecting the node pair (i,j) can also be denoted by a single index, say l.

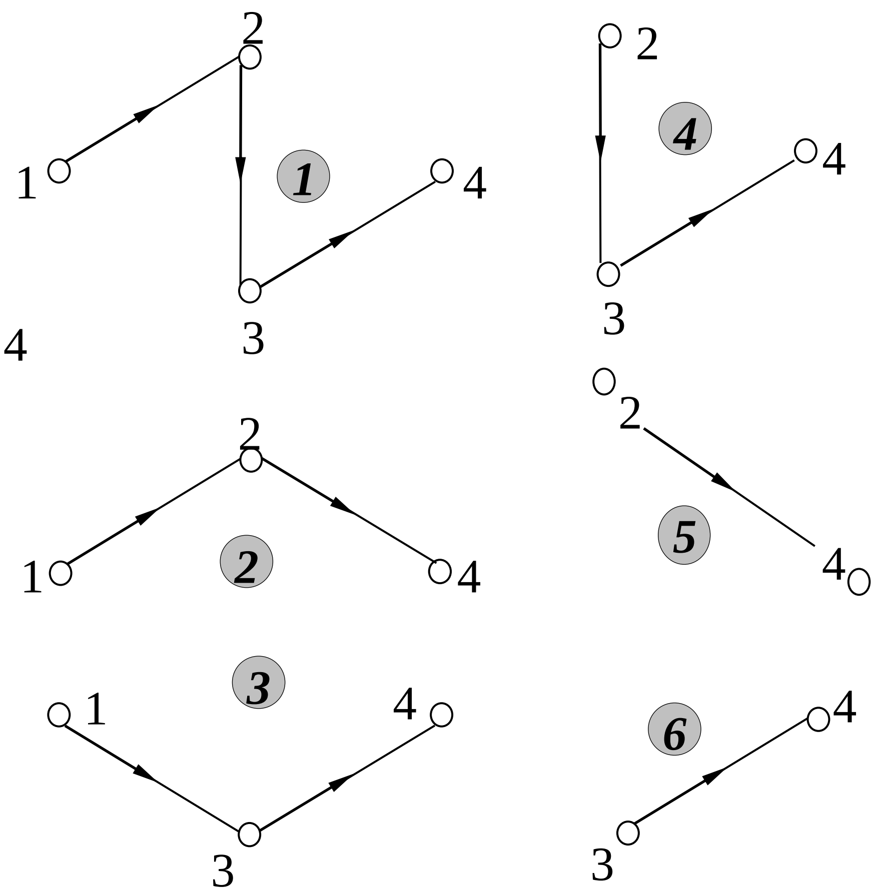
Path: sequence of consecutive links connecting an initial node (path origin) to a final node (path destination).

Graph theory

GRAPH



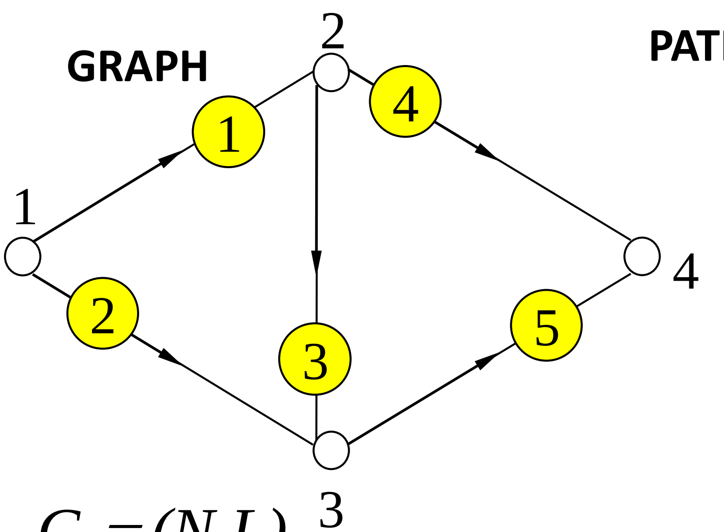
PATHS



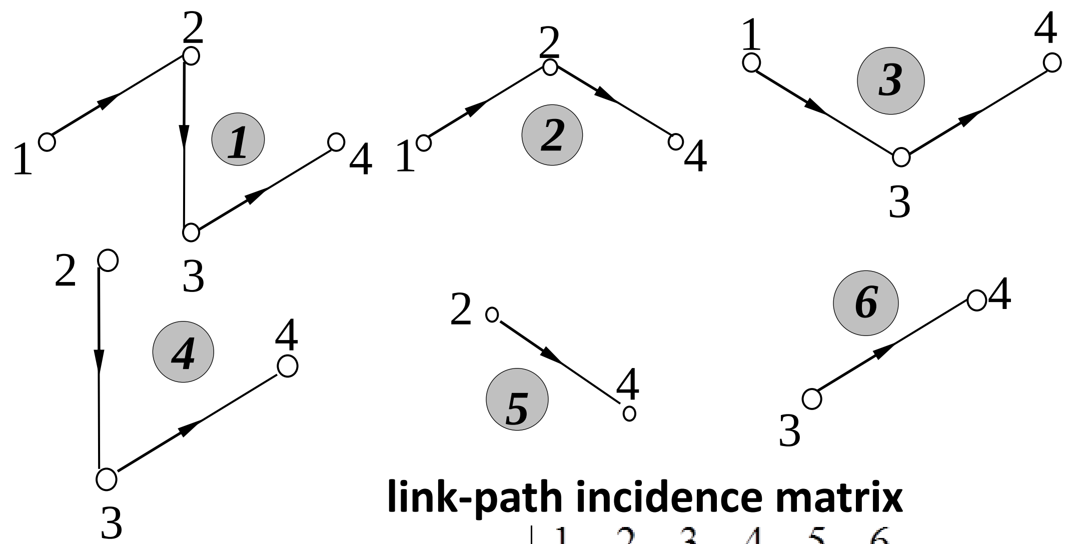
Link-path incidence matrix

Matrix whose generic element $a_{i,j}$ is *1* if link *i* belongs to path *j*, *0* otherwise

GRAPH



PATHS



$G \equiv (N,L)$

$N \equiv \{(1,2,3,4)\}$

$L \equiv \{(1,2),(1,3),(2,3),(2,4),(3,4)\}$

Origin centroids $\{1, 2, 3\}$

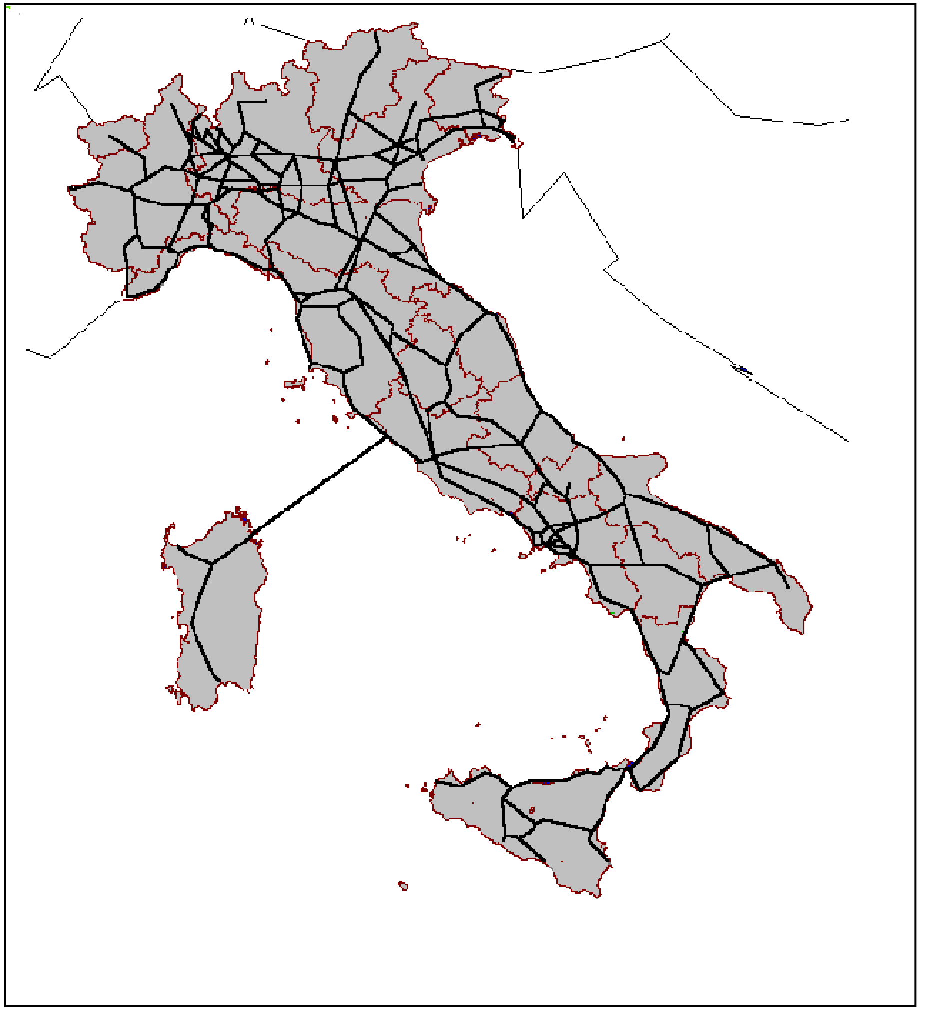
Destination centroid $\{4\}$

link-path incidence matrix

$$\underline{A} = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1(1,2) & 1 & 1 & 0 & 0 & 0 & 0 \\ 2(1,3) & 0 & 0 & 1 & 0 & 0 & 0 \\ 3(2,3) & 1 & 0 & 0 & 1 & 0 & 0 \\ 4(2,4) & 0 & 1 & 0 & 0 & 1 & 0 \\ 5(3,4) & 1 & 0 & 1 & 1 & 0 & 1 \end{array}$$

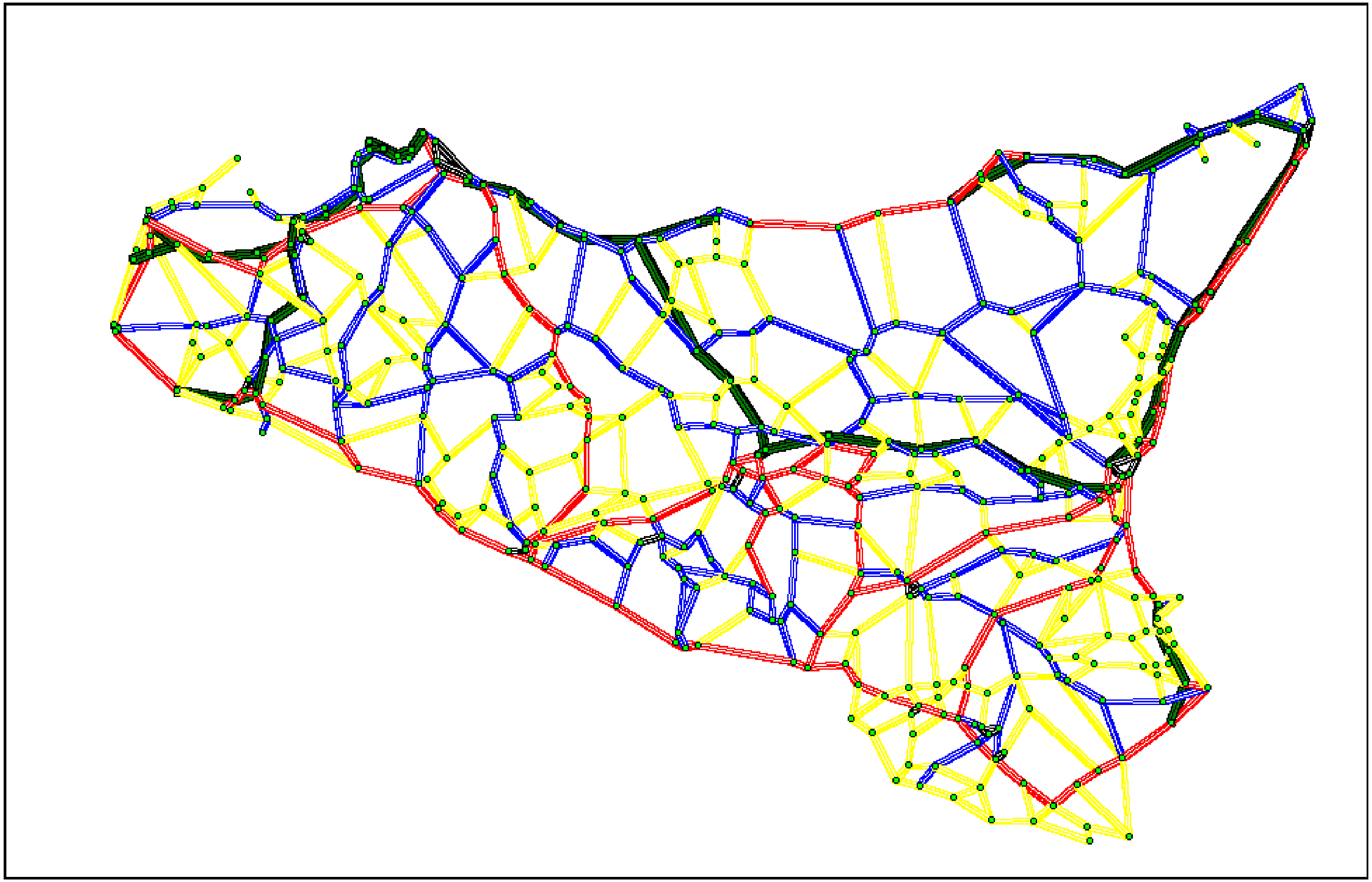
Examples

Graph of national railways infrastructures



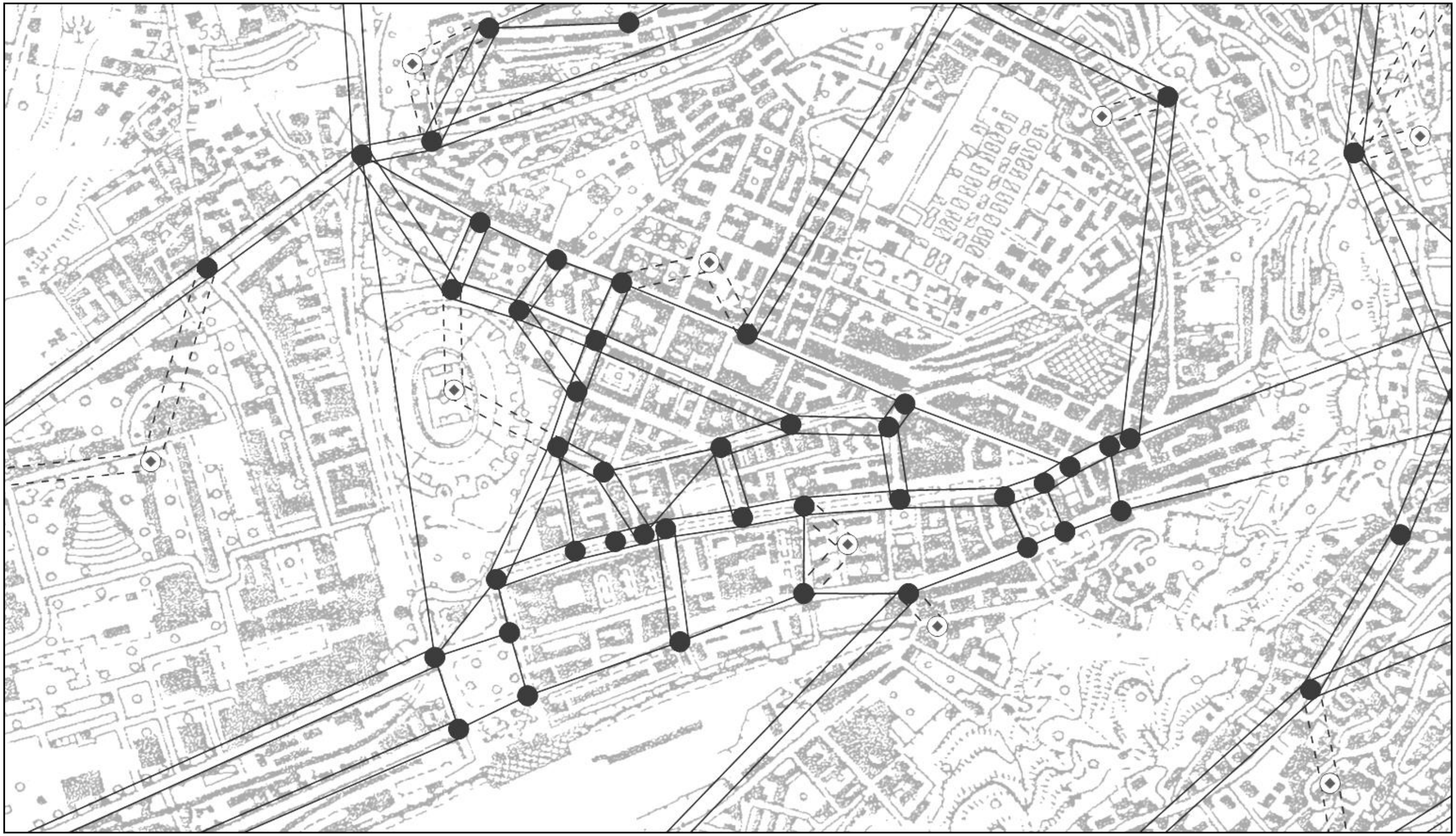
Examples

Graph of regional roads



Examples

Graph of urban roads



Transport networks

- Network:
  graph whose elements are associated with a quantitative characteristic

- MONO-MODAL NETWORKS
  - road network
  - public transport network
    - infrastructure network
    - service network

- MULTI-MODAL NETWORKS
  - all modes and services together

Link characteristics

- **Infrastructure networks** (e.g. roads, rail,….)
  - length
  - geometric characteristics
  - design speed
  - …
- **Service Networks** (air, bus, rail, maritime)
  - travel times
  - waiting times
  - fares
  - …

Generalised costs

LINK COST:

$$c_l = \beta_1 \, t_l + \beta_2 \; cm_l$$

where:

- c_l is the generalized transport cost related to link l
- t_l is the travel time for crossing link l
- cm_l is the monetary cost (e.g. the toll) related to link l
- β_1 and β_2 reciprocal substitution coefficients

Generalised costs

PATH COST:

$$C_k = \sum_{l \in k} c_l = \sum_{l} a_{lk} c_l \longrightarrow \underline{C} = \underline{A}^T \underline{c}$$

where:

C_k is the generalized transport cost related to path *k*

c_l is the generalized transport cost related to link *l* belonging to path *k*

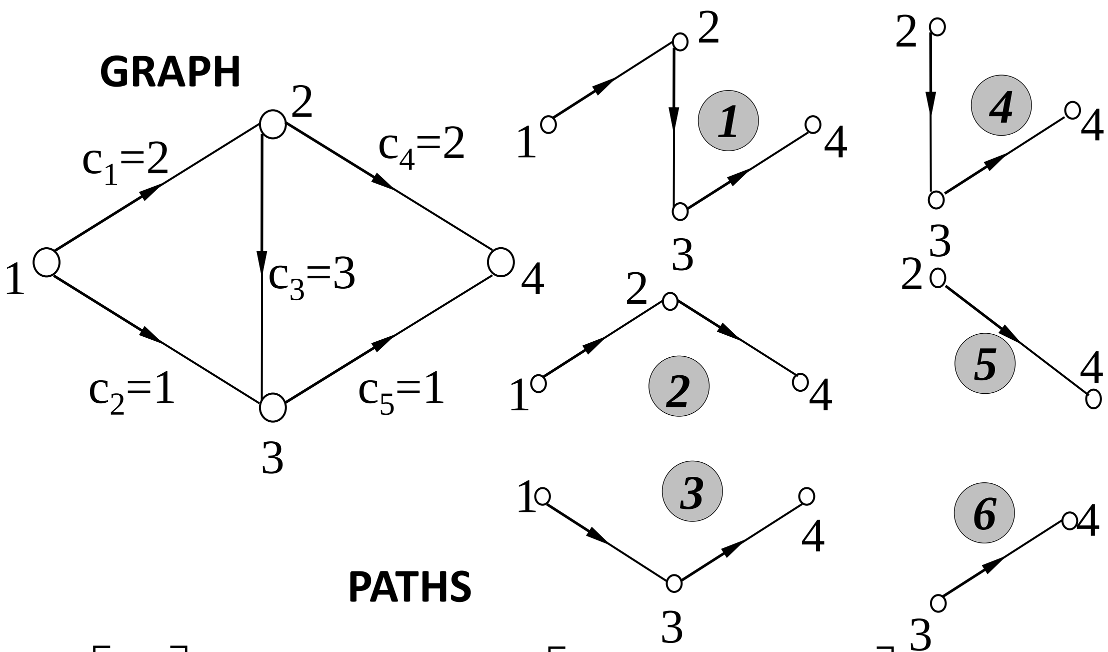
a_{lk} is equal to *1* if the link *l* belongs to the path *k*, *0* otherwise

$\underline{A}$ link-path incidence matrix

$\underline{C}$ path generalized cost vector

$\underline{c}$ link generalized cost vector

Example



link-path incidence matrix

$\underline{A} =$

	1	2	3	4	5	6
1(1,2)	1	1	0	0	0	0
2(1,3)	0	0	1	0	0	0
3(2,3)	1	0	0	1	0	0
4(2,4)	0	1	0	0	1	0
5(3,4)	1	0	1	1	0	1

$$\underline{\mathbf{C}} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} = \underline{\mathbf{C}}^{ADD} = \underline{\mathbf{A}}^T \cdot \underline{\mathbf{c}} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} c_{12} + c_{23} + c_{34} \\ c_{12} + c_{24} \\ c_{13} + c_{34} \\ c_{23} + c_{34} \\ c_{24} \\ c_{34} \end{bmatrix}$$

$$\underset{\underline{C}}{\begin{bmatrix} 6 \\ 4 \\ 2 \\ 4 \\ 2 \\ 1 \end{bmatrix}} = \underset{\underline{A}^T}{\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}} \cdot \underset{\underline{c}}{\begin{bmatrix} 2 \\ 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}}$$

Flows

LINK FLOW $\underline{f}$:

number of users (e.g. vehicles) crossing link *l* in the time unit

PATH FLOW $\underline{F}$:

number of users (e.g. vehicles) following path *k* in the time unit

$$f_l = \sum_k a_{lk} F_k \longrightarrow \underline{f} = \underline{A}\,\underline{F}$$

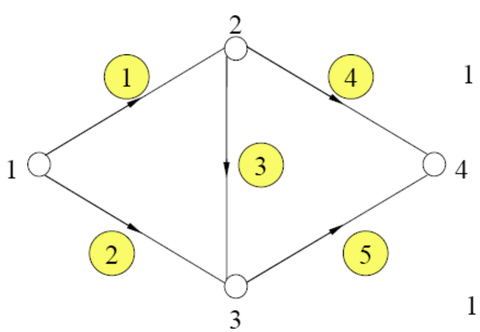
where:

- f_l is the flow on link *l*
- a_{lk} is equal to *1* if the link *l* belongs to the path *k*, *0* otherwise
- F_k is the flow on path *k*
- $\underline{f}$ is the link flow vector
- $\underline{F}$ is the path flow vector
- $\underline{A}$ is the link-path incidence matrix

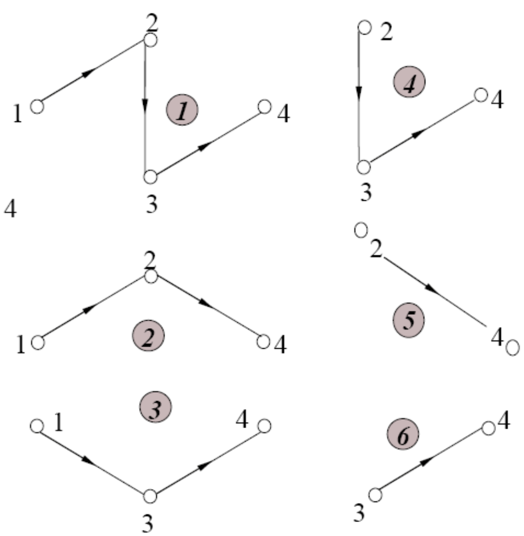
Example

$$\underline{f} = \begin{bmatrix} f_{12} \\ f_{13} \\ f_{23} \\ f_{24} \\ f_{34} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix} = \underline{A} \cdot \underline{F} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} F_1 + F_2 \\ F_3 \\ F_1 + F_4 \\ F_2 + F_5 \\ F_1 + F_3 + F_4 + F_6 \end{bmatrix}$$

GRAFO



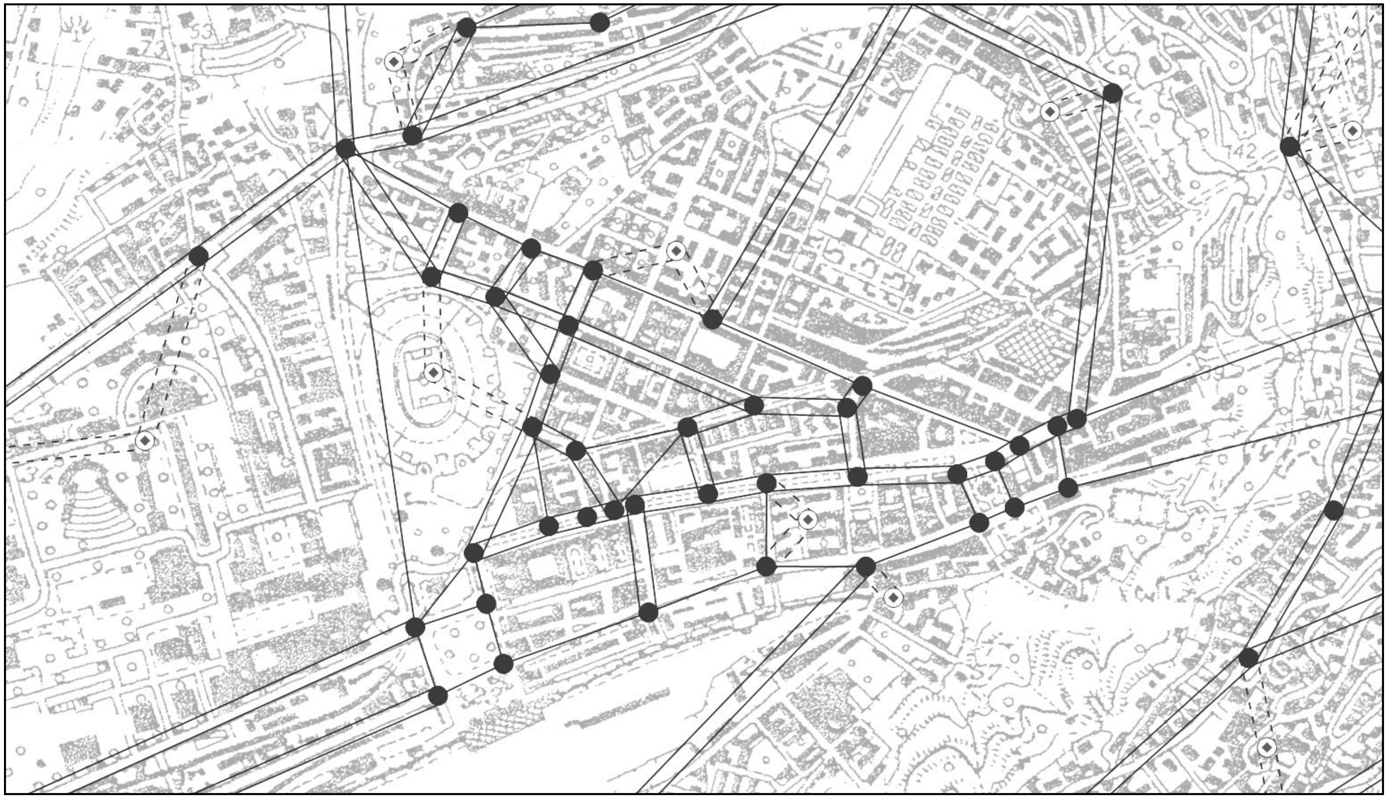
PERCORSI



$$\begin{bmatrix} 133 \\ 867 \\ 195 \\ 1439 \\ 1861 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 16 \\ 117 \\ 867 \\ 179 \\ 1321 \\ 800 \end{bmatrix}$$

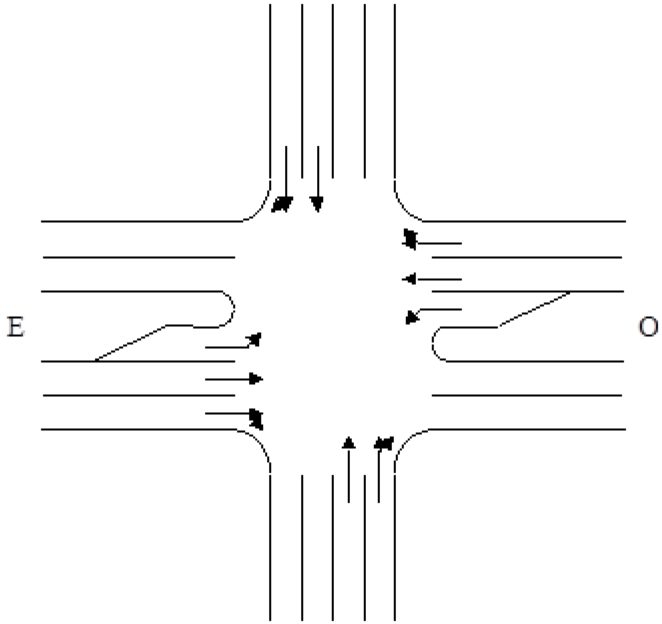
Urban road graph

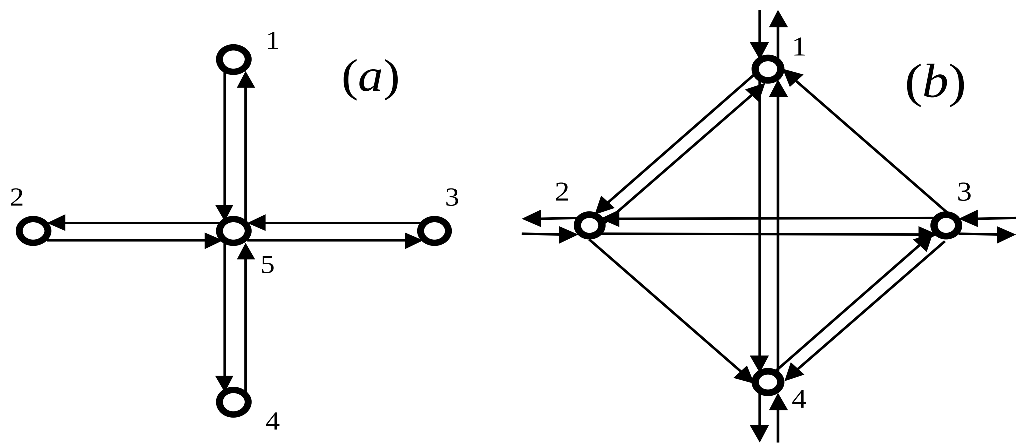
- ✓ nodes usually represent intersections
- ✓ links usually correspond to connections between nodes allowed by the circulation scheme



Urban road graph

- ✓ the level of detail depends on the scope of the simulation
- ✓ the same element can be represented by different graphs





a) single node representation
b) detailed representation

Transit (Public Transport) graph

Transit systems are different from private cars because they are non-continuous both in:

- ✓ space

  as services accessibility is possible only in some points (***stops***)

- ✓ time

  as services accessibility is possible only in defined time periods (according to the ***timetable***)

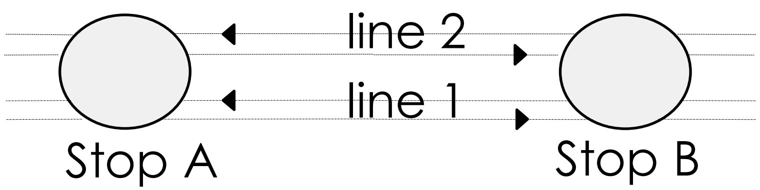
Transit graph

Transit graphs are more complex than road ones as they have to represent non-continuity and intermodality (e.g. at least pedestrian access/egress to stops and stop-to-stop on-board travelling and possible vehicle changes at interchanges)

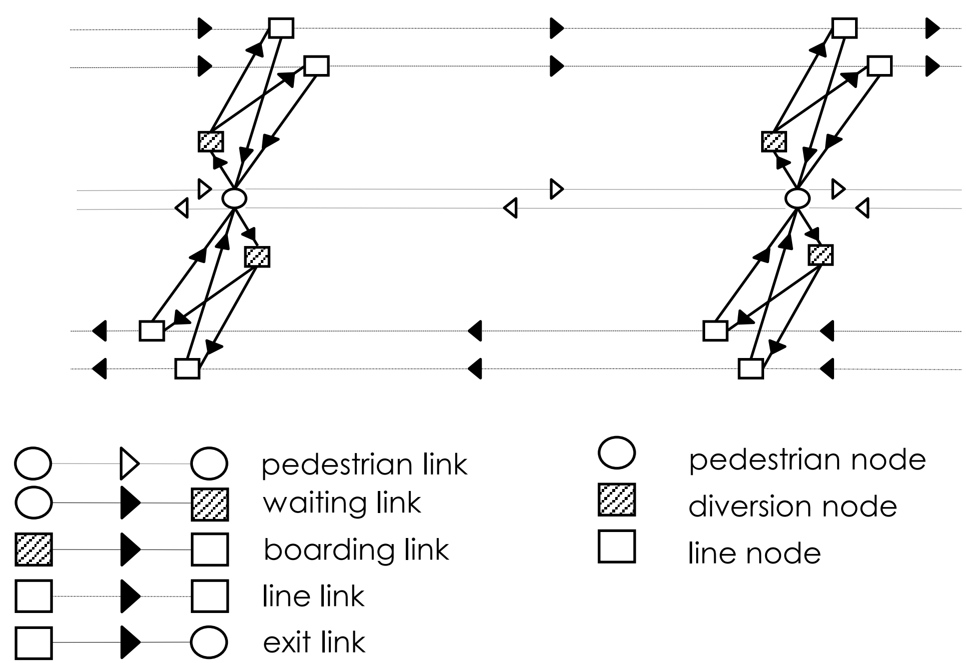
LINE-BASED APPROACH

(graph of lines)

BASE SCHEME



REPRESENTATIVE GRAPH



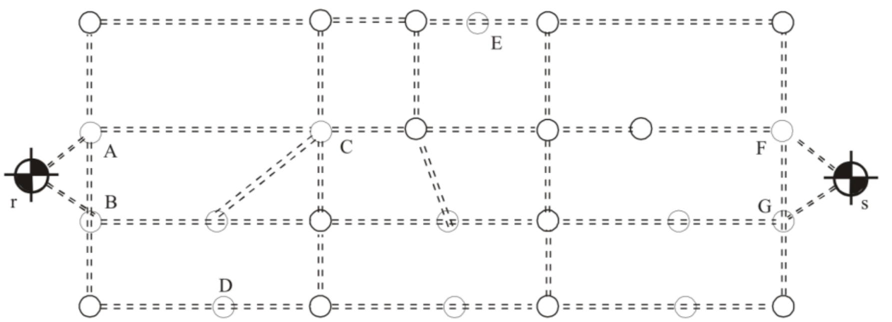
Transit graph

Line-based approach

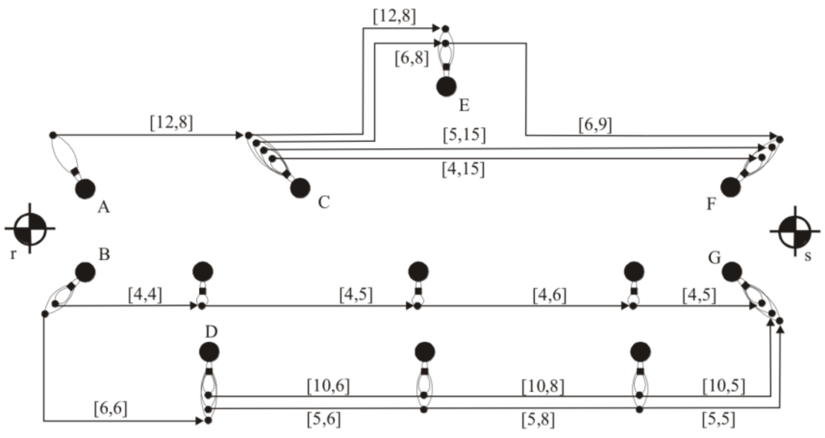
Transit graphs are usually made of :

- ✓ an access-egress sub-graph
- ✓ service (lines) sub-graph

access-egress sub-graph

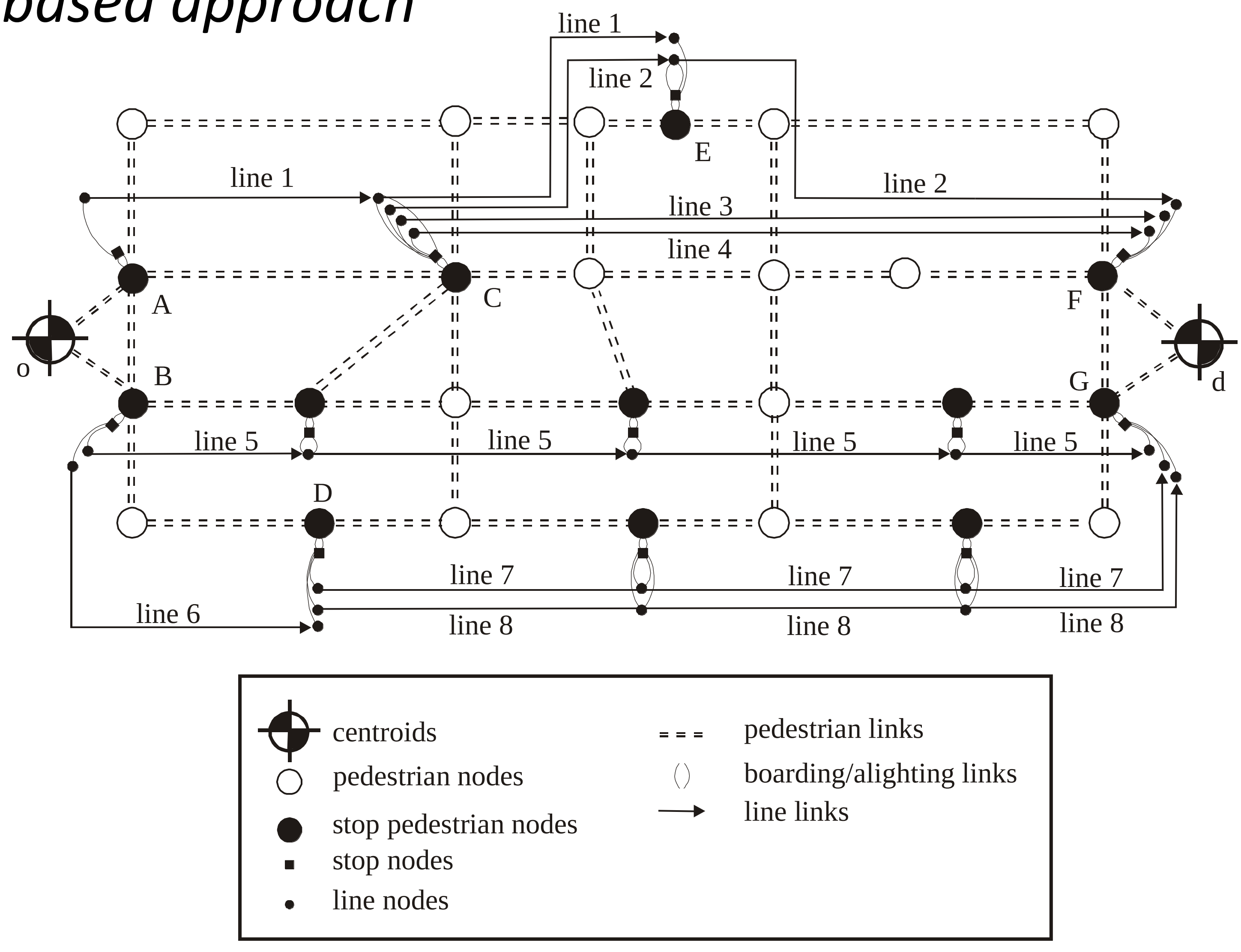


service (lines) sub-graph

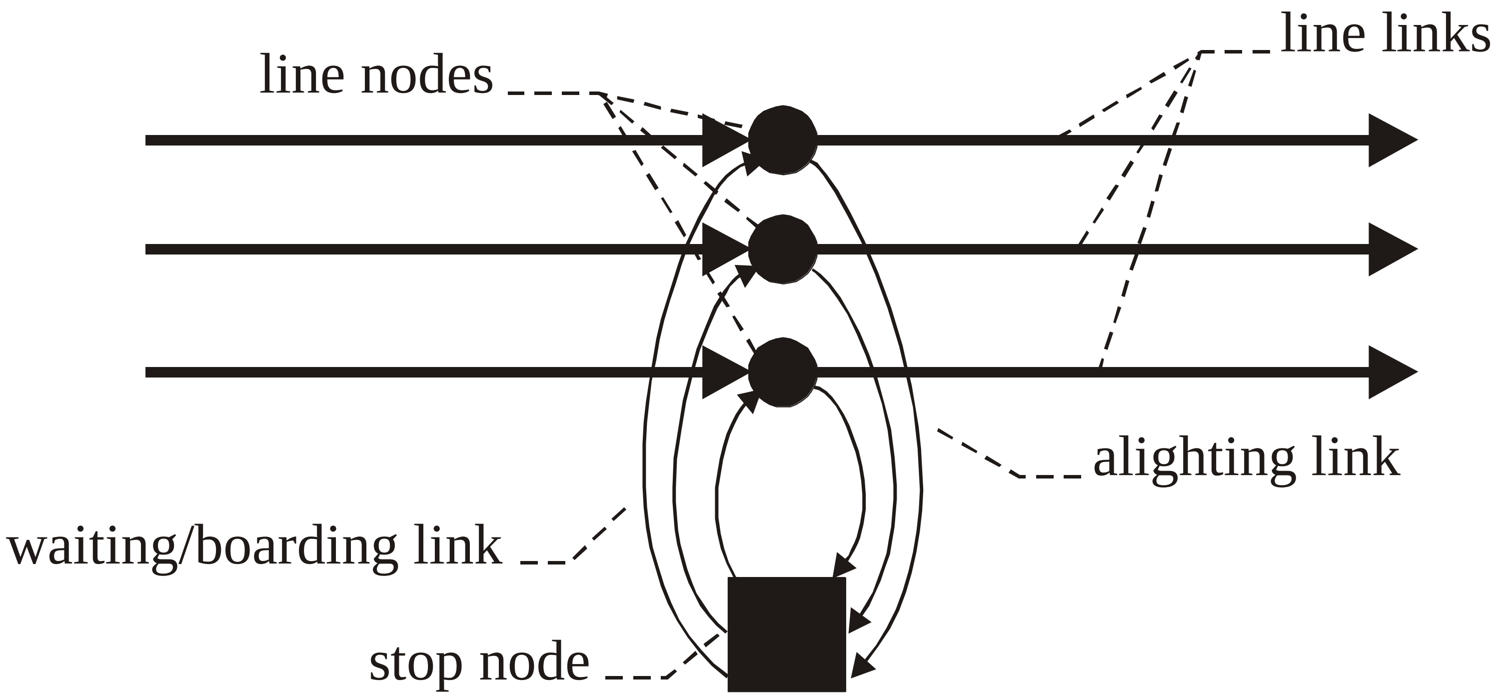


Transit graph

Line-based approach



Transit graph
Line-based approach



Cost functions

Cost functions give the generalised cost connected to the use of the generic link *l*, which is perceived by the user and influences his/her mobility choices.

Cost functions usually depend on the link flow as follows:

$$c_l(f) = cv_l(f) + co_l$$

with:

$cv_l(f)$: variable cost (e.g. travel time and/or waiting time)

co_l : fixed cost (e.g. tolls)

Cost functions for road links

Transport cost of a road link can be sub-divided into three components:

- running time (to cover the link);
- waiting time (at ending junctions, at toll gates, …);
- monetary cost.

$$c_l\,(\underline{f}) = \beta_1\, tr_l(\underline{f}) + \beta_2\, tw_l(\underline{f}) + \beta_3\, cm_l(\underline{f})$$

with:

$tr_l(\underline{f})$ running time of link *l* depending on flow vector

$tw_l(\underline{f})$ waiting time on link *l* depending on flow link

$cm_l(\underline{f})$ monetary cost of link *l* depending on flow link

i.e. $cm_l = c_{toll} + c_{fuel}(\underline{f})$

β_1 , β_2 , β_3 homogenization parameters

Example of cost function

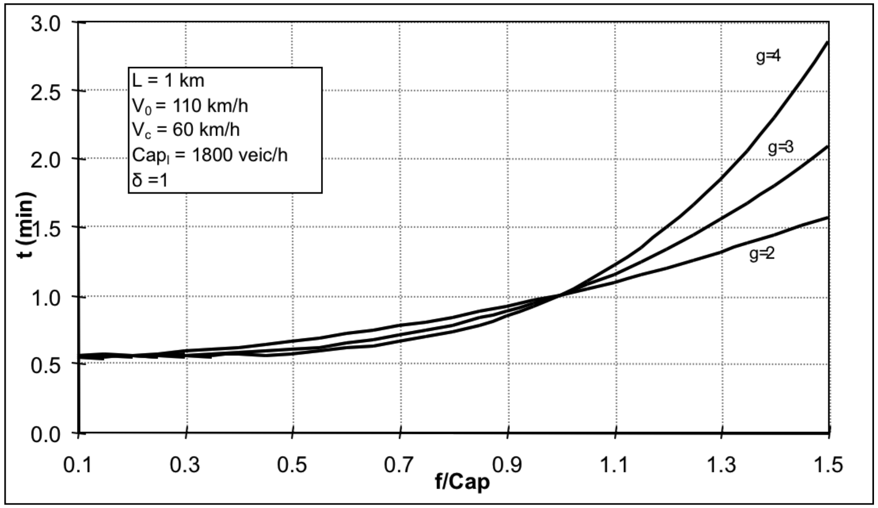
Motorway links

Uninterrupted flow conditions (the waiting component can be ignored)

$$tr_l(f_l) = \frac{L_l}{V_o} + \delta\left(\frac{L_l}{V_c} - \frac{L_l}{V_o}\right)\left(\frac{f_l}{Cap_l}\right)^{\gamma}$$

with:

- L_l = length of link *l*
- V_o = average free flow speed
- V_c = average speed with flow equal to capacity
- Cap_l = capacity of link *l*, (e.g. $Cap_l = Nlanes_l \cdot Cap_u$)
- δ and γ function parameters



Example of cost function

Extra-urban links (2 lanes roadway)

Uninterrupted flow conditions (the waiting component can be ignored)

$$tr_l(f_l) = \frac{L_l}{V_o} + \delta\left(\frac{L_l}{V_c} - \frac{L_l}{V_o}\right)\left(\frac{f_l}{Cap_l}\right)^{\gamma}$$

with:

L_l = length of link *l*

V_o = average free flow speed

V_c = average speed with flow equal to capacity

Cap_l = total capacity of link *l*, i.e. $Cap_l = N_{lanes} \cdot Cap_u$

δ and γ function parameters

Example of cost function

Extra-urban links (2 lanes roadway)

Uninterrupted flow conditions (the waiting component can be ignored)

$$V_o = 56.6 + 3.2\,L_u + 4.5\,L_o - 2.4\,P - 9.6T - 5.4D \quad (km/h)$$

with:

L_u width of the link (m)

L_o distance of lateral obstacles from road side (m)

P slope of the link (%)

T winding degree of link (high = 1, null = 0)

D lateral bother coefficient (1 if present , 0 otherwise)

Example of cost function

Extra-urban links (1 lanes roadway)

Uninterrupted flow conditions (the waiting component can be ignored)

$$t_{r_l}(f_l, f_{l*}) = \frac{L_l}{V_o} + \delta\left(\frac{L_l}{V_c} - \frac{L_l}{V_o}\right)\left(\frac{f_l + f_{l*}}{Cap_{ll*}}\right)^{\gamma}$$

with:

$l*$	link representing the opposite roadway to the considered one
Cap_{ll*}	global capacity of both ways
V_o (km/h)	$56.6 + 3.2\, L_u + 4.5\, L_o - 2.4\, P - 9.6T - 5.4D$

At first approach the following values can be considered :

$V_c = 40\text{-}45$ km/h; $Cap_{ll*} = 2000\text{-}2600$ veic/h; $\gamma = 3$; $\delta = 1$

Example of cost function

Toll gate links

running time = mean waiting time (queue theory)

$$tw_l(f_l) = T_s + (T_s^2 + \sigma_s^2).\frac{f_l}{2}.\frac{1}{1 - f_l/Cap_l} \qquad f_l \leq \alpha\, Cap_l$$

$$tw_l(f_l) = tw_l(\alpha Cap_l) + \frac{d}{df} tw_l(f)\Big|_{f=\alpha cap_l} (f_l - \alpha Cap_l) \qquad f_l > \alpha\, Cap_l$$

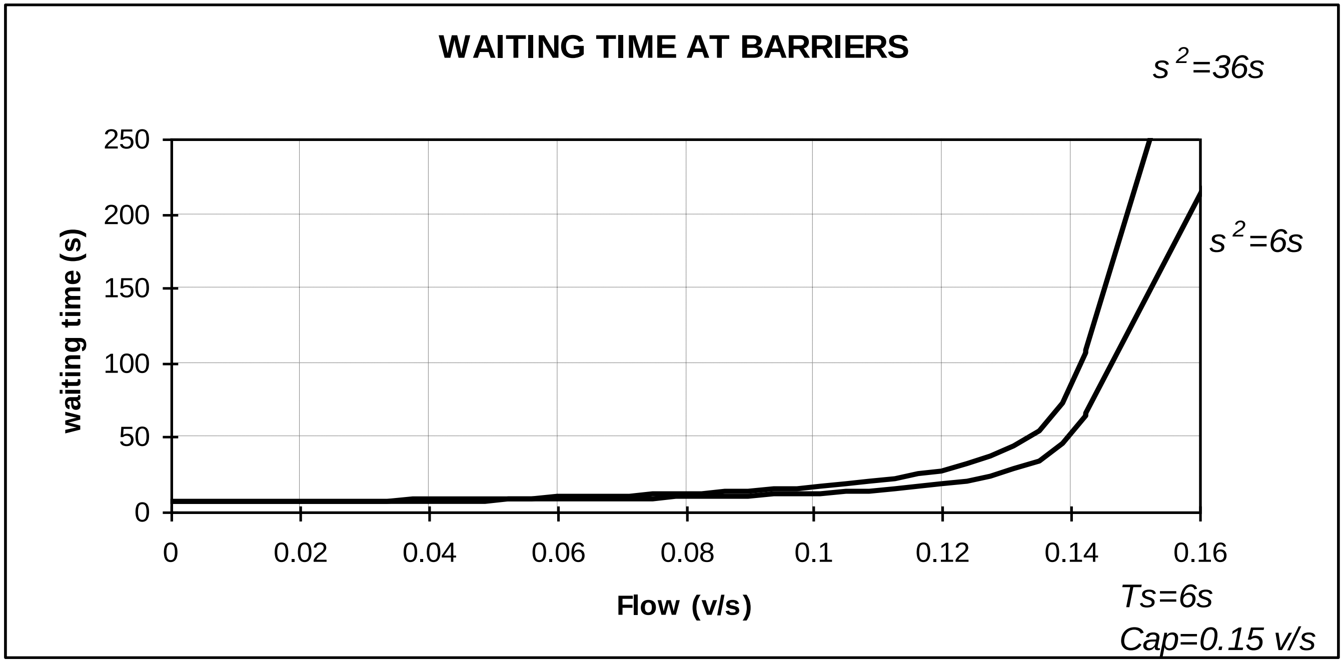
with:

T_s	mean operation time at gate
$\sigma^2{}_s$	variance of operation time at gate
$Cap_l = N_{gate}/T_s$	capacity of the link (barrier) equal to the number of gates (N_{gate}) multiplied by the capacity of a gate $(1/T_s)$

Example of cost function

Toll gate links

running time = mean waiting time (queue theory)



$$tw_l(f_l) = T_s + (T_s^2 + \sigma_s^2) \cdot \frac{f_l}{2} \cdot \frac{1}{1 - f_l / Cap_l} \qquad f_l \leq \alpha\, Cap_l$$

$$tw_l(f_l) = tw_l(\alpha Cap_l) + \frac{d}{df} tw_l(f)\Big|_{f = \alpha cap_l} (f_l - \alpha Cap_l) \qquad f_l > \alpha\, Cap_l$$

Example of cost function

Urban road links

Such links are characterised by:

- ✓ short link length (some hundreds of meters)
- ✓ running speed barely influenced by flow, (short distance between junctions, speed limits)
- ✓ significant waiting time at junctions

running time tr_l

$$tr_l(f_l) = \frac{L_l}{V_l(f_l)}$$

with:

L_l link length

V_l average running speed of link

f_l flow on the link

Example of cost function

Urban road links

average running speed V_l

$$V_l\,(km/h) = 31.1 + 2.8\,Lu_l - 1.2\,P_l - 12.8\,T_l - 10.4 D_l$$
$$- 1.4\,INT - [0.000053 + 0.000123\,X]\,(f_l / Lu_l)^2$$

with:

Lu	roadway width, once deducted the space occupied by parking [m]
P	mean slope in percentage unit (%)
T	winding degree of link [0,1]
D	circulation bother degree [0,1]
INT	number of secondary junctions per kilometer
X	shadow variable = 1 if overtaking is not allowed; = 0 otherwise
f	Link flow [veh/h].

Example of cost function

Urban road links

delay at signalized junctions tw_l

(single junctions with traffic light with no reserved lane)

$$tw_l = tw_l(X_l) \quad \text{with} \quad X_l = f_l / cap_l$$

with:

tw_l waiting time of link *l*

X_l saturation degree of link *l* at final section

S_l saturation flow of link *l* at final section, equal to the maximum number of vehicles that could pass if the light is continuously green (*μ = 1).*

μ green fraction

G time length of effective green

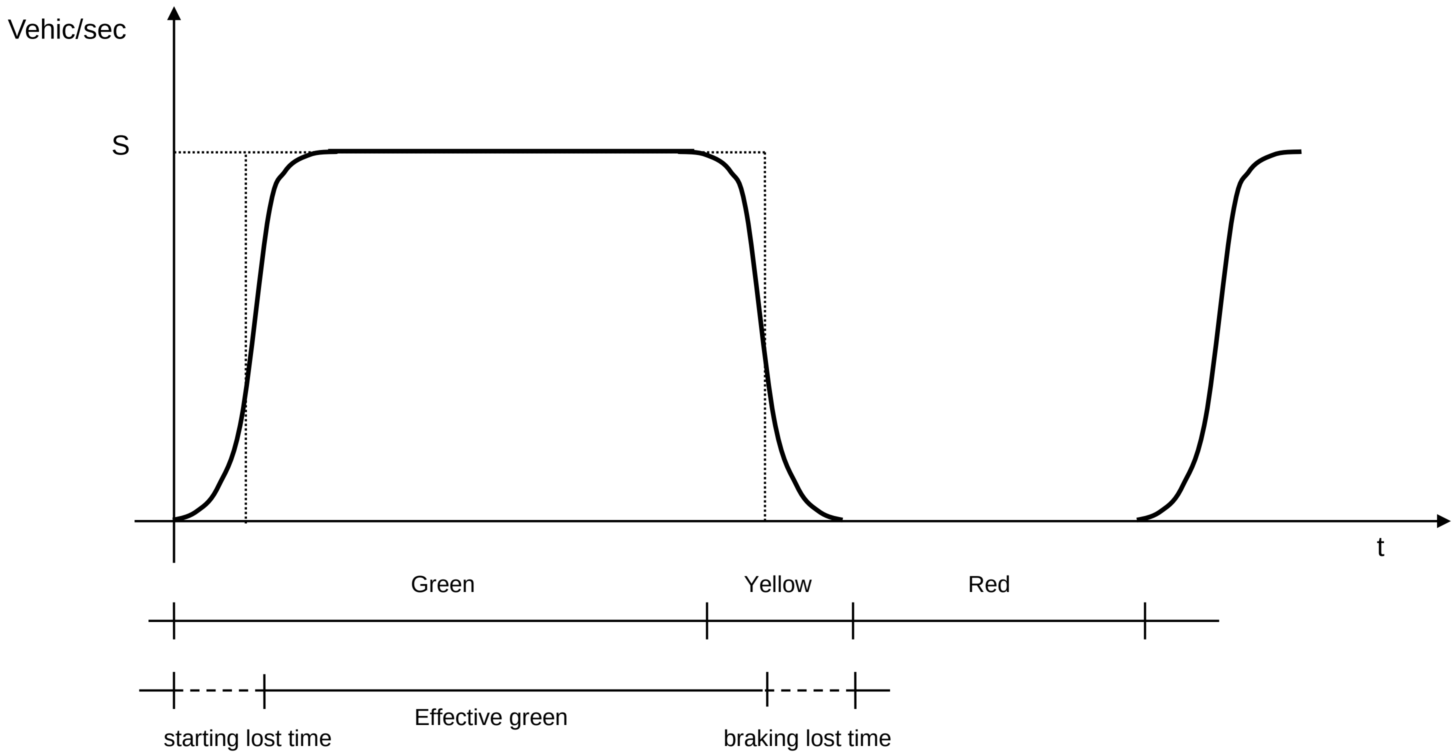
T_c time length of traffic light cycle

Example of cost function

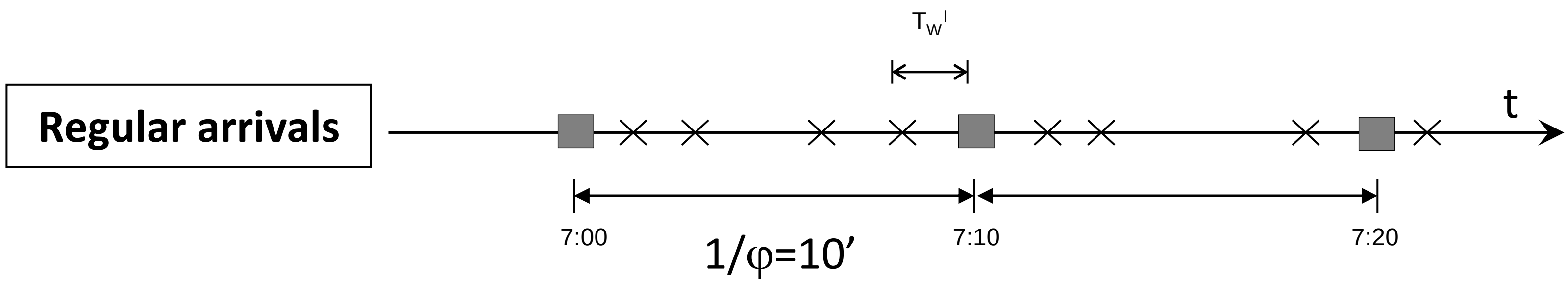
Urban road links

delay at signalized junctions tw_l

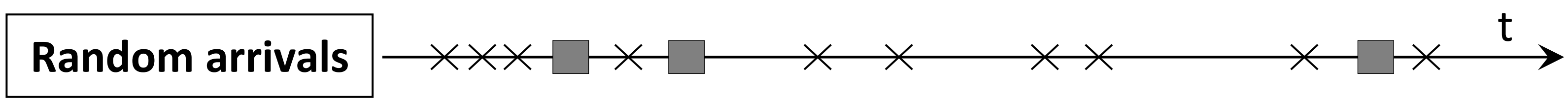
(single junctions with traffic light with no reserved lane)



Cost functions for transit
waiting time



$$E[t_w^i] = t_w = 10'/2 = 5'$$



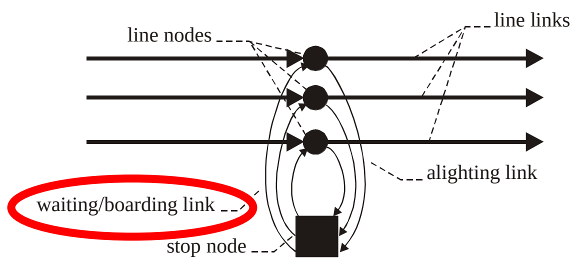
$$E[t_w^i] = t_w = 10'$$

Bus arrival time at bus stop

User arrival time at bus stop

Frequency $\varphi = 1/10'$

Cost functions for transit
waiting time



only one line available: $$tw_l = \frac{\theta}{\varphi_l}$$

more *"attractive" lines*: $$tw_l = \frac{\theta}{\varphi_L} \quad \text{where} \quad \varphi_L = \sum_{l \in L} \varphi_l$$

with

tw_l average waiting time for line *l*

φ_l frequency of line *l* (number of runs/time unit)

θ = 0.5 for regular line arrivals

= 1 for random line arrivals

Performance and impact functions

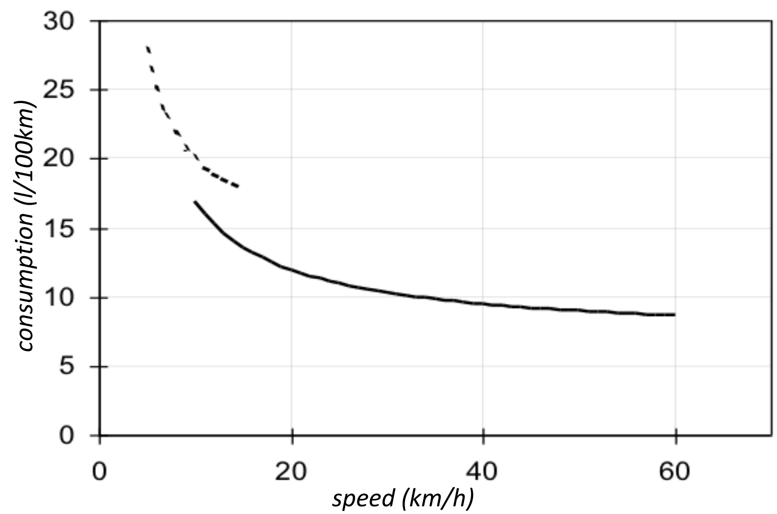
Mathematical functions relating performances and impacts to physical and functional parameters of the specific transportation systems and, in some cases, to link flows

Performance functions:

“internal” effects borne by the users, but not perceived in their mobility choices (e.g. indirect vehicle costs as consumptions, accident risks,…)

Impact functions:

effects “external” to the transportation system (e.g. noise and air pollution,…)



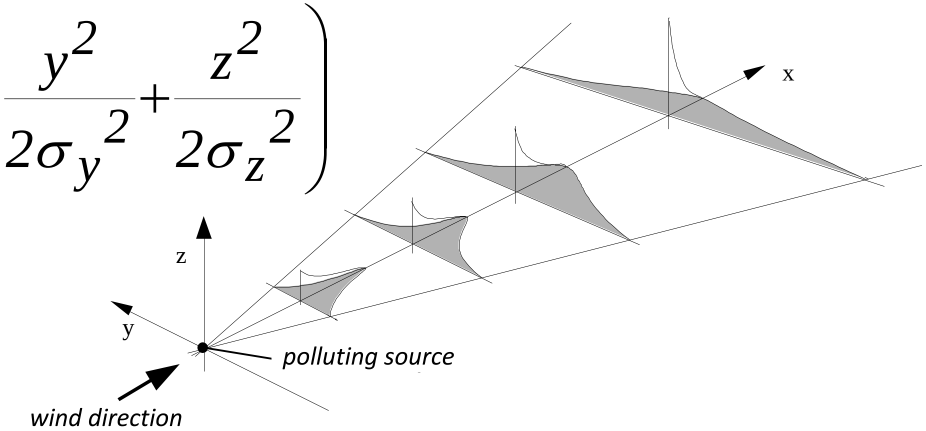
——— $C = C_R + \dfrac{800}{V_M\left(V_M + 8\right)}$ *for* $V_M < 15 km/h$

- - - - - - $C = 7.0 + \dfrac{99}{V_M}$ *for* $10 \le V_M \le 60\ km/h$

Examples of impact functions

Dispersion of air Pollutant: Gaussian models

$$C(E,u,x,y,z)=\frac{E}{u}\cdot\frac{1}{2\pi\sigma_y\sigma_z}e^{-\left(\frac{y^2}{2\sigma_y^2}+\frac{z^2}{2\sigma_z^2}\right)}$$



with:
o origin of the polluting source;
axis *x* = horizontal axis coinciding with the wind direction;
axis *y* = horizontal axis perpendicular to the axis *x*;
axis *z* = vertical axis perpendicular to *x*-*y* plane;
C = concentration of pollutant at point *x,y,z (g/mc)*;
E = pollutant emission from the source per time unit (*g/sec)*;
u = average wind speed *(m/sec)*;
s_y,s_z = dispersion coefficients =*s* (*x*, *st*) (*m)*;
x = distance along the wind direction (*m)*;
st = atmospheric stability class.

Examples of impact functions

Noise pollution

Noise level simulation:

"Sydney" model (1977)

$$L_{10} = 56 + 10.7\, ln\, f - 18.5\, ln\, d + 0.3\, p \qquad [dB(A)]$$

$$L_{eq} = 55.5 + 10.2\, ln\, f - 19.3\, ln\, d + 0.3\, p \qquad [dB(A)]$$

where L_{eq} is the equivalent noise level, f is the hourly link flow (summing up flows per direction in case of two-way streets), d is the distance (*m*) between from the edge of the carriageway and the road center line and *p* is the heavy vehicle percentage (experimental field 0≤*p*≤35%), weighting >3.5t; uninterrupted flow conditions.

Examples of impact functions

Noise pollution

Noise level simulation:

"Reggio Calabria" model (1991)

$$L_{eq} = 52.78 + 5.20\, ln\, (f_{eq}/d) + 0.68\, V$$

where L_{eq} is the equivalent noise level, f_{eq} is the link flow in equivalent vehicles (*veic/h*) with passenger car equivalent coefficient for trucks equal to *6*, V is the average speed (*km/h)* and d is the distance (m) from the edge of the carriageway. Such a model was calibrated for an urban case with a U-building layout.

Building a supply model

a) Delimitation of the study area

b) Zoning

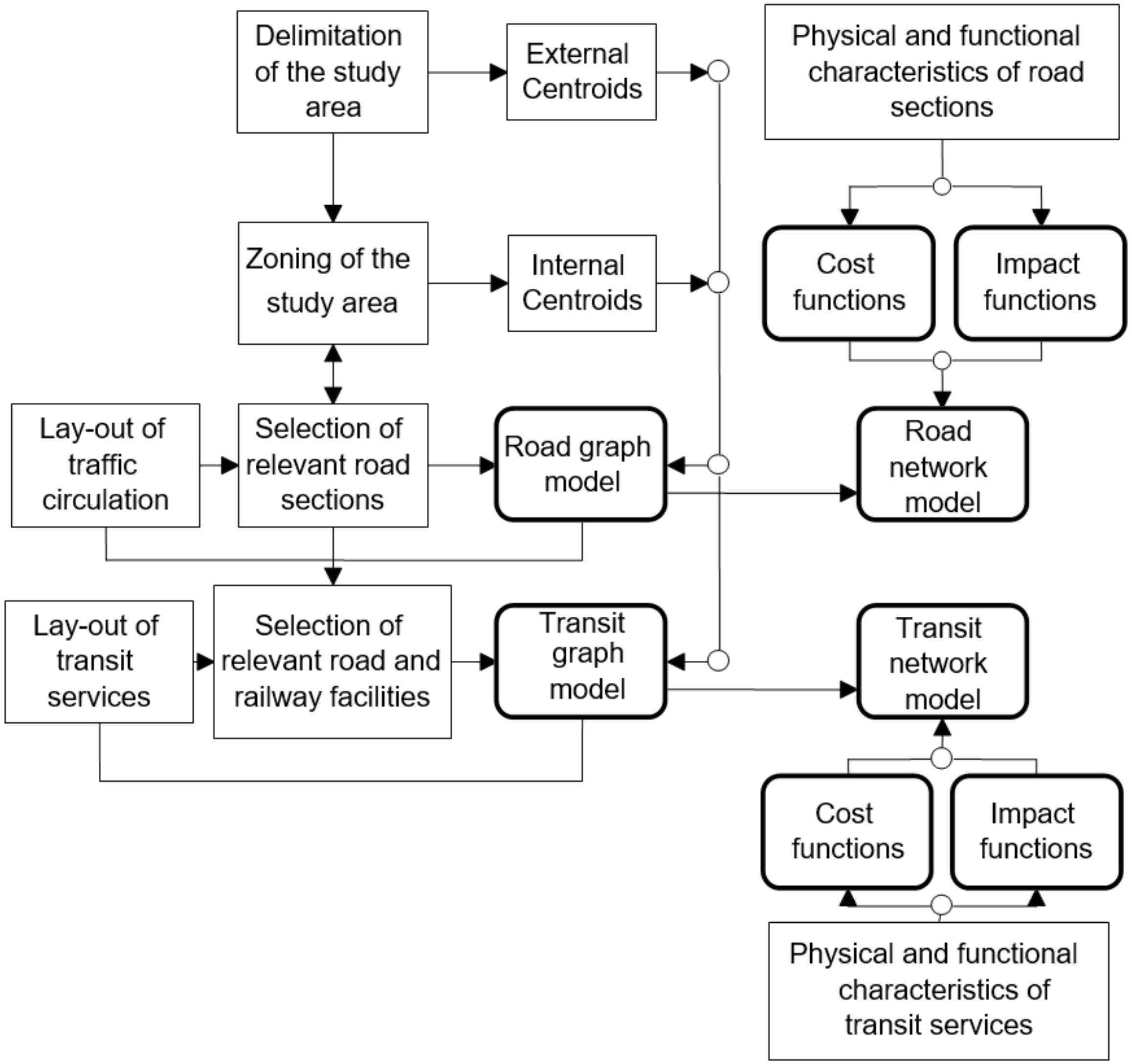
c) Selection of relevant supply elements (basic network)

d) Graph construction

e) Identification of performance and cost functions

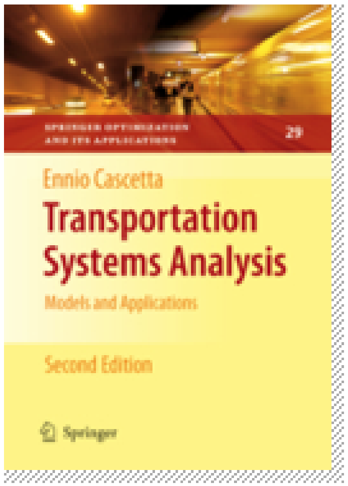
f) Identification of impact functions

Building a supply model



Reading materials

http://www.springer.com/mathematics/applications/book/978-0-387-75856-5



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Lecturer references

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